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*Full Length Research Paper*

# Evolutionary processes in the development of errors in subtraction algorithms

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**The study of errors made in subtraction is a research subject approached from different theoretical premises that affect different components of the algorithmic process as triggers of their generation. In the following research an attempt has been made to investigate the typology and nature of errors which occur in subtractions and their evolution throughout primary education. The main aim of the research is to examine whether in our school context systematic errors are made and if these decrease throughout schooling. In order to do this, we carried out a rigorous analysis of over 7140 subtractions done by children of 7 to 13 years of age in 2nd, 3rd, 4th, 5th and 6th years of primary school, using a sample of 357 primary school students who were tested with the VanLehn 20 subtractions test. The SPSS 11.5 computer programme was used to analyze the data generated by the tests. One hundred and twenty-two different errors were analyzed, the results showing systematic errors in 55% of the cases. We likewise found that the evolution of the error throughout primary education shows certain similarities with the results obtained in other teaching contexts. The results obtained are undoubtedly valuable for programming the teaching process.**

**Key words:** Algorithmic learning, primary education, errors in subtraction, empirical research, transverse research on subtraction error, systematic error.

## INTRODUCTION

A revision of the most relevant literature dealing with errors in algorithmic processes highlights the fact that systematic errors produced during learning are ana-lyzed from two theoretical perspectives centred around the semantics or the syntax (Resnick, 1982) of the acquisition of the skill. The first line of research in which, amongst others, we find authors such as (Car-penter and Moser, 1984; Carpenter et al., 1996; De Corte and Verschaffel, 1987; Fuson, 1986, 1992; Fuson and Briars, 1990; Hiebert and Lefevre, 1986; Nesher et al., 1982; Ohlsson and Rees, 1991; Resnick, 1982, 1983; Resnick and Omanson, 1987; Sander and Richard, 1997; Sander, 2001), has focused on the study of the conceptual background that children acquire dur-ing the learning process of multiple column sub trac-tion algorithms. The syntactic approach, on the other hand, related to the VanLehn Theory, has contributed interesting data on the procedural mechanisms that govern the generation of systematic errors (Brown and Burton, 1978; Brown and VanLehn, 1980, 1982; VanLehn, 1982, 1983, 1987, 1990; Young and O´Shea, 1981). In this context, and without going into the classic dichotomy of concepts v. processes, the research which this article is based on tries to confirm the contributions of the procedural or the syntactic perspectives as defined by Resnick (1982) in the specific context of our country.

With regard to the subject at hand, the most relevant results can be found in the U.S.A, where from the seventies onwards, in the setting of cognitive research, interesting scientific contributions have been made from a procedural perspective. This line of investigation, lead by authors such as Brown and Burton (1978); Brown and VanLehn (1982); VanLehn (1982, 1983, 1990); Young and O´Shea (1981), illustrates that some students showed erroneous processes, "*buggy proced-*

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**Table 1.** $N_{20}$ = Total number of students who correctly completed the 20 subtractions distributed by school year.

| Year | N sample | N 20 | % over the sample total in each year |
|---|---|---|---|
| Second | 64 | 4 | 6.25 % |
| Third | 72 | 15 | 20.83 % |
| Fourth | 73 | 23 | 31.51 % |
| Fifth | 75 | 30 | 40.00 % |
| Sixth | 73 | 23 | 31.51 % |
| Total | 357 | 95 | 26.61 % |

*ures – Buggy algorithmic*", (Brown and Burton, 1978; VanLehn, 1982, 1990), ingrained in the cognitive area of subtraction resolution. Therefore, according to the Repair Theory, such initial errors produced during example-based induction learning of subtraction bec-ome systematic and end up forming a part of the procedure (Brown and VanLehn, 1980; VanLehn, 1982, 1983, 1990).

In the following research set out in this article we have tried to confirm the existence and typology of such arithmetic "bugs" in the context of schools in our country.

## RESEARCH AIMS AND METHODOLOGY

The aims of this phase of the research were:

i). To analyze the typology of the errors most frequently made by school children from the 2nd to the 6th year of primary education.
ii). To observe the evolution of these errors throughout the different years in primary education.
iii). To analyze and compare the results with those obtained by reference authors in this field of research.

### Subjects

The 20 subtraction test of VanLehn (1990) was given to a comprehensive sample of 357 subjects from 2nd, 3rd, 4th, 5th and 6th years of primary education, of ages ranging from 7 to 12 years old.

### Context

The context in which the research was carried out was a province in the west of Spain. In general, the economic resources of this province come from agriculture and livestock-raising, and there is a high degree of rural population drift.

A sample of 357 primary school students (58, 50%, boys / 41, 50%, girls), between the ages of 7 and 12 were tested with the VanLehn 20 subtractions test, (VanLehn, 1990). This test is comprised of 20 multicolumn subtractions, seventeen of which are subtractions with borrowings. According to the author, this test has been carefully designed in order to obtain different errors, (VanLehn1990).

For the sample we took all students from the 2nd, 3rd, 4th, 5th, and 6th years of four primary schools, two of which are located in the city of Salamanca and the other two located in rural areas of the province. The students from the city schools come from upper-middle, lower-middle and lower class families. The stud-Ents from the rural schools, located in a mountainous region, are from middle and lower socio-economic backgrounds.

### Procedure

To analyze the data base generated by the tests we used the SPSS 11.5 computer programme. Data analysis consisted mainly of using descriptive statistical techniques adapted to the nature of the variables studied. The overall descriptive statistical analysis employed frequency analysis, measures of central tendency and dispersion, according to each case, and analysis of the corresponding figures. One hundred and twenty-two different types of errors were analyzed out of a total of 7140 subtractions.

The first drawback we encountered was establishing categories which would allow us to group together in an orderly fashion the data at our disposal. As theoretical-practical analysis precedents already existed, we chose to take as a reference the categories created by Brown and Burton (1978), Brown and VanLehn (1982) and VanLehn, (1990). The established analysis categories were:
(i) Right answers on the test, and (ii) Errors. Within the Errors category we included the following subcategories: "Bug"," Bug-free" and "non- diagnosable". In order to define error typology we used the Vanlehn (1990) glossary of errors. The methodological procedure followed can be summarised in the following steps:

(i) The VanLehn (1990) test was given to 357 children aged between 7 and 12.
(ii) The 7140 subtractions on the test were then corrected and the errors classified.
(iii) The errors were categorized according to the bugs identified by Vanlehn (1990)
(iv) Error typology was analyzed using SPSS 11.5.

## RESULTS

### Study of the cases that correctly answered all 20 subtractions in the VanLehn (1990) test

From the total sample population (n = 357), 26.61% correctly answered all 20 subtractions. The highest number of right answers occurred in school (3), which is closely related to the best socio-contextual conditions from the outset (middle-high class socioeconomic level) . The distribution of right answers per year can be seen in Table 1.

As can be seen in the above Table, the percentage of correct answers increases over the different years until the fifth year, where they drop to the same frequencies as those for the fourth year. In our view, the influence of a non-spiral and therefore decontextualised curriculum in the mathematics field naturally affects the decline in algorithmic information significantly from the fourth year onwards, the number of errors starting again to increase from the fifth year onwards.

### Study of the errors made in the VanLehn, (1990) test

The error percentage was 23.47%. The number of errors drops linearly by years, with a proportionality coefficient equal to (-73.8), showing a stability in the fall between the fifth and the seventh years. The greatest numbers of errors were concentrated in the following

**Table 2.** Frequency and percentage of errors by years n= 357.

| Errors | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Total |
|---|---|---|---|---|---|---|
| Frequency | 60 | 57 | 50 | 45 | 50 | 262 |
| Percentage | (22.9%) | (21.8%) | (19. 1 %) | (17.2%) | (19.1 %) | (73.38%) |

**Table 3.** Frequency of appearance of "calculation error" by years.

| No . appearances/Year | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|
| 0 | 33 | 45 | 48 | 53 | 53 | 232 |
| 1 | 23 | 27 | 20 | 15 | 14 | 89 |
| 2 | 4 | 6 | 4 | 6 | 5 | 25 |
| 3 | 4 | 2 | 1 | 1 | 0 | 8 |
| 4 | 0 | 2 | 0 | 0 | 0 | 2 |
| 6 | 0 | 0 | 0 | 0 | 1 | 1 |
| Total | 64 | 72 | 73 | 75 | 73 | 357 |

subtractions according to the following percentages: Subtraction në.13 (1813-215), 31.37%/ Subtraction në. 16 (4015-607), 28.39%/Subtraction në 17 (702-108), 31.37%/ Subtraction në. 18 (2006-42), 32.49%/ Subtrac-tion në 19 (10012-214), 37.53%/ Subtraction në 20 (8001-34), 33.89%.

This grouping together of errors in these subtractions could be due to the conceptual structure underlying them. The handling of rules inherent to the transformation of the zero is fundamentally derived from this structure.

Finally, the error frequency with regard to the population sample taken by year can be seen in the following Table 2.

From the analysis of the previous Table, we conclude that the frequency of errors, with regard to the sample by year, drops in the fifth year only to increase again in the sixth. According to Brown and Burton (1978), this outcome may be affected by the fact that in the teaching process the addition and subtraction algorithms are not worked on after year 3.

Analysis of the errors yielded the following results: (i) 51.3% of the tests analyzed showed more than one bug or type of error, (ii) the error which appeared with most frequency was the "calculation error" and (iii) some of the errors with greatest concentration in frequencies that persisted throughout the years were therefore systematic in nature.

With regard to the most frequent error, "*calculation errors*" comprised 35.01% of the total. This can be seen in Table 3, where we show the frequency of error appearance in relation to the number of subjects committing the error by years

The percentage obtained with regard to the "non-diagnosable" category was 5.88%.

In order to determine whether the errors that appeared in our research could be found in each child individually and partially or dominantly, we adopted a methodological position that consisted of analyzing the errors that grouped together the greatest frequencies and that were repeated throughout all of the years, and considered as dominant the appearance of the same type of error on more than three occasions over the total of 20 subtractions per child.

The number of occurrences and the number of children that made systematic errors in years 3, 4, 5 and 6 are shown in the following Table.

Although combined errors occurred, they are not included in the previous Table. The first five errors (in italics) appeared predominantly and not partially in the individual tests.

On the other hand, some bugs appeared with a high frequency rate, but cannot be considered stable due to the methodological criteria that we have chosen, given that they disappear in the fourth year. This is the case for the "Smaller-from-larger" or "Stop-borrow-at-zero" bugs. This fact would support the speculation that there are errors of a semantic nature that disappear with instruction and give way to procedural errors. With regard to this type of error, in the following Table we can see the incidence of those which have a high frequency during the first phase of algorithm acquisition in the second and third year but disappear with instruction.

Therefore, we find in the results a greater percentage of children with stable bugs in all the years and also a lesser decrease of these by year, although there is a proportionality in the decrease between fourth and fifth years, which are subject to the influence of a greater number of children categorized as "error-free", as shown in the following Table.

## DISCUSSION

The first of the assertions that led to this research re-

revolved around the study of errors from a situational perspective focusing on our country. In this sense, the results obtained in this phase of the research show that in our classrooms subtraction algorithm errors occur and that in 55.5% of the cases these are stable throughout the whole sample.

On the other hand, we notice that the frequency of errors per year decreases, showing a point of inflection in the fifth year before increasing in the sixth year to results comparable to those obtained in the fourth year. This trend was also observed by Brown and Burton (1978), who reported similar results throughout the different years and a percentage of errors by year equal to those obtained in our research.

We likewise found a typology of systematic errors similar to that found by other important authors who have dealt with the subject (Young and O´Shea, 1981; Brown and Vanlehn, 1982; Vanlehn, 1990), conciding with these three studies in two errors, these being "*Borrow-from-zero* and *Borrow-across-zero*". With regard to the "*Smaller -from-larger*" and "*Stops- borrow-at-zero"* "*bugs*", these appear with a very high frequency but disappear from the fourth year onwards as a result of instruction.

The previous Tables show that the errors revolve around behaviour that systematically affects the phases of greater cognitive complexity of the process and relate directly to the comprehension of concepts essential to the learning of the algorithm. These are essentially the sphere of principles that govern the decimal numerals system.

Therefore, is it possible that a common evolutionary line exists in the acquisition of errors during the learning of subtractions that could explain the appearance of similar errors in different teaching-learning contexts? The results provided could obviously raise this possibility.

If we analyze the evolution of the errors via these results, we find that 51.3% of the cases studied show more than one bug or type of error. The percentages of errors drop linearly by year with a proportional coefficient of (-73.8) as the level increases, although in the sixth year a lesser drop occurs owing to the phenomenon of a fall in algorithmic information.

Of those error categories which are most prevalent, "*calculation errors*" show the greatest frequency. Their percentage of appearance is 35.01% throughout the sample, which is comparable, although not specifically, to the results obtained in the VanLehn (1990) research, with a 37% appearance of the same error, and to those obtained by Young and O´Shea (1981) with 27.13%. VanLehn (1990), reports that the proportion of cases that demonstrate this error category decreases in his sample with instruction, diminishing as the level increased.

In the same way, in Table 3, we establish how the appearance of instances of these types of errors in the sample decreases in the higher years, as they do in the research taken as reference. We can likewise report that the errors of 5.88% of the students formed part of the "non-diagnosable" category.

VanLehn (1990) again investigated those children who committed errors assigned to the "*non-diagnosable* and *calculation errors*" categories and concluded that if the tests had been carried out twice, the number of students would have been reduced by a quarter. He thus illustrated that these errors which appeared in a constant manner, throughout all the years, were due to a "*noise*" in the human information processor, and although the error was constant throughout the years, it was not sufficient to be able to establish a diagnosis. We consider this author's argument as an explanation which is overly linked to psychology given that more concrete and real causes exist, essentially of a pedagogic-situational nature, that depend on the classroom context, its characteristics and the moment of the tests' execution.

We therefore assess these errors from our perspective and define them as "*accumulator errors* ", as their origin is diverse and could be based both on causes of a procedural origin (such as scarce training in numerical calculation, given that they are in fact produced with greater frequency in the 2nd and 3rd year) and causes linked to attitudes related to the context in which the test is carried out. The lack of concentration, motivation, or experience in these types of test, etc., can be attitude-related causes that have a bearing. These causes would explain their appearance through-out all the years but with a tendency to decrease as the school level increases.

On analyzing the results we observed that the students, as a general rule, presented more than one systematic error in their answers. Such findings have also been found in other studies (Brown and VanLehn, 1980; Young and O´Shea, 1981; VanLehn, 1982, 1990).

As we have been able to see in Table 4, some of the errors with a greater concentration of frequencies persisted in certain types of subtraction throughout years 2 to 6. These are what we have defined in our research as "*systematic errors of a stable nature*"; others disappear after the fourth year.

We find in the results a greater percentage of children with stable bugs in all years and also a lesser decrease in them by year, although we can appreciate a proportionality in the decrease between fourth and fifth year, which are subject to the influence of a greater number of children categorized as "*error-free*".

In his research in 1982, VanLehn found specifically that stable bugs were present in 49% of students in the third year, 27% of students in the fourth year and 13% of those in the fifth year, concluding that the differences between years were due to the fact that the older children had learnt the correct algorithm. The evolution of the percentages was clear: 19% of students in the third year, 39% of the students in the fourth year and 60% of the students in the fifth year were in the "*bug free*" cate-

**Table 4.** Years 3, 4, 5, 6. No. of occurrences and children who consistently showed signs of the bug.

| Error name and category* | No. occurrences | No. children |
|---|---|---|
| *1-1=0-after-borrow* | 61 | 13 |
| *Borrow-no-decrement* | 48 | 22 |
| *Borrow-from-zero-is-ten* | 38 | 8 |
| *Borrow-from-at-zero* | 28 | 13 |
| *Forget-borrow-over-blank* | 19 | 3 |
| *Diff, 0-N=N* | 13 | 8 |
| Borrow-no-decrement-except last | 9 | 4 |
| Always-borrow | 6 | 3 |
| Add-instead-off-sub | 4 | 1 |
| Borrow-into-one=ten | 3 | 1 |
| Borrow-across-zero | 3 | 1 |
| Always borrow-left | 3 | 1 |
| Ignore-left-most-one-over-blank | 2 | 1 |

We took as a referent only years 3, 4, 5 and 6, since in year 2 the complete algorithm of subtraction with borrowing has not been fully taught.

**Table 5.** Errors with greatest frequency in the first phase of subtraction algorithm acquisition. 2nd and 3rd Year.

| Error name and category | 2nd | | 3rd | |
|---|---|---|---|---|
| | Frequency | % | Frequency | % |
| Smaller-from-larger | 119 | 18.36 | 20 | 5.20 |
| Calculation error | 44 | 18.82 | 35 | 9.11 |
| Borrow-from-zero-is-ten | 19 | 2.93 | 39 | 10.15 |
| Add-borrow-decrement | 18 | 2.77 | 0 | 0 |
| Don’t decrement-zero-over-zero | 18 | 2.77 | 0 | 0 |
| Diff, 0-N=0 | 16 | 2.46 | 6 | 1.56 |
| Add-instead-of-sub | 14 | 1.38 | 1 | 0.26 |
| Dic, N-0=0 | 13 | 2 | 11 | 2.86 |
| Don´t decrement-zero-over-blank | 12 | 1.85 | 3 | 0.78 |
| Borrow-from-zero | 10 | 1.54 | 15 | 3.90 |

gory. In our research, these percentages are shown in Table 6. Nevertheless, although the results cannot be specifically compared, given the characteristics of the different research studies, in our sample we found that 55.55% of students in the third year, 52.05% of those in the fourth year and 26.66% in the fifth year showed signs of stable bugs compared with the percentages of children categorized as “error free”; 23.61% in third year, 23.28% in fourth year and 33.33% in the fifth year. We compared the percentages of stable errors found in our research with those found by VanLehn (1990) , and confirm the existence of a greater percentage of children with stable bugs in our sample, in all years, and also a lesser decrease by years. Nevertheless, we observed proportionality in the drop between fourth and fifth years, which are subject to the influence of a greater number of children categorized as error free and with a greater number of right answers.

The greatest numbers of errors (37.53%) were concentrated in subtraction no. 19: (10012-214), whose conceptual structure is characterised by some specific traits that define its conceptual architecture and therefore its procedural mechanism.

We also confirm the existence of the majority of “bugs” found by authors that we have used as referents. Two of these - Borrow-across-zero and Borrow-from-zero- appear in the work of Brown and VanLehn (1980); Young and O´Shea (1981); VanLehn (1990), both of which are related to the transformation of zero. We also confirm the existence of other “bugs” that appear with very high frequency in our research, in the same way as they do in other studies, but which we cannot consider stable, probably due to the methodological criteria that we have chosen. This is the case for errors such as

**Table 6**. Percentages of children distributed according to categories and years.

| Category | 3rd Year, (8-9 years old) | 4th Year (9-10 years old) | 5th Year (10-11 years old) | 6th Year (11-12 years old) |
|---|---|---|---|---|
| Error | 55.55% | 52.05% | 26.66% | 32.61% |
| Error-free | 23.61% | 23.28% | 33.33% | 32.87% |
| Right answer | 20.83% | 31.55% | 40.0% | 31.55% |

Smaller-from-larger or Stop-borrow-at-zero. This supports the speculation that there are errors of a semantic nature that disappear and give way to those of a procedural nature. In general, we can point out that the comparison with other theories cannot be carried out exhaustively given that the contexts, resources and samples are different. Nevertheless, taking as a reference Table 7.17 from Vanlehn (1990), which includes the systematic bugs found in the studies by Young and O´Shea (1981), Brown and Vanlehn (1982) and Vanlehn (1990), we can report that the appearance of the systematic bugs "Borrow-*across-zero, and Borrow-from-zero*" are present in the results of all four studies, and both bugs are closely related to the transformation of zero

## Conclusions

The results described here show that in our classrooms, systematic errors are made in 55.5% of the cases throughout the primary years. On a didactic level, finding these errors and recognizing the most frequent typology is undoubtedly of educational value, since it allows teachers to act preventively with regard to these errors.

It should also be pointed out that we have found a typology and an evolution in the acquisition of systematic errors similar to those found by important authors in the literature who have approached the topic and whose research has been carried out in other educational contexts. It is therefore possible that there is a common evolutionary line in the acquisition of errors when learning subtraction that would explain the appearance of similar errors in different teaching contexts.

If this is so, teaching of the subtraction algorithm could be programmed in such a way that it considers the existence of this evolution in the acquisition of errors in order to avoid its appearance.

In conclusion, we believe that the contributions made by Brown and Burton (1978), Brown and VanLehn (1982) VanLehn and Brown (1980), VanLehn, (1982, 1983, 1987, 1990), Young and O´Shea (1981) can be generalised to very different temporal and spatial didactic contexts. We acknowledge an evolutionary similarity in what we could define as acquisition of errors, and therefore consider that the conclusions obtained in the study of these are of incalculable value in helping teachers who teach algorithmic processes in the classroom environment. For this reason, we believe that they can help understand the origin of errors, their evolution and improve the diagnosis of such as a preventative measure in primary school teaching.

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