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Full Length Research Paper

Designing a group decision support system under uncertainty using group Fuzzy analytic network process (ANP)

Wei Zhong Yang[1]*, Yu Hui Ge[1], Jian Jia He[1] and Bin Liu[2]

[1]School of Management, University of Shanghai for Science and Technology Shanghai, P. R. China, 200093.
[2]Nuclear Energy Economics and Management Research Centre, University of South China, Hengyang, P. R. China, 421000.

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Handling uncertainty in decision making is recently receiving considerable attention by researchers. Advances in group Fuzzy analytic network process (ANP) are discussed to support decision making because of the complexity and vagueness under uncertainty. An adaptive group Fuzzy ANP group decision support system (DSS) under uncertainty is put forth that makes up for some deficiencies in the conventional ANP. Fuzzy judgments are firstly used when it is difficult to characterize the uncertainty by point-valued judgments due to partially known information, and a bipartite graph is formulated to model the problem of group decision making under uncertainty. Then, a Fuzzy prioritization method is proposed to derive the local priorities from missing or inconsistent Fuzzy pairwise comparison judgments. As a result of the unlikeliness for all the decision makers to evaluate all elements under uncertainty, an original aggregation method is developed to cope with the situation where some of the local priorities are missing. Finally, an evaluation of petroleum contaminated site remedial countermeasures using the proposed group Fuzzy ANP, indicates that the presented group DSS can effectively handle uncertainty and support group decision making with high level of user satisfaction.

Key words: Group decision support, uncertainty, analytic network process, group Fuzzy ANP, aggregation, site remedial countermeasures.

INTRODUCTION

From the dialectical point of view, uncertainty is absolute, and certainty is relative (Sahistein and Erin, 2006). Uncertainty is an essential component of every day life, and has become an important characteristic of modern deci-sion support systems (DSS). There are many decision making methods proposed by various authors to model uncertainty (Yager, 2004; Ekel et al., 2008; Zarghami and Szidarovszky, 2009; Hosseini et al., 2010; Mishalani and Gong, 2009; Reneke, 2009). However, it is hard to find any general definition of uncertainty in these literatures on ‘uncertainty’ modeling. In decision logic, the definition of uncertainty by Zimmermann (2000) is generally accepted as a standard. He focuses on the human- related, subjective interpretation of uncertainty, which implies that in a certain situation, a person does not dispose information, which is quantitatively and quaitatively appro-priate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics. In this sense, uncertainty could be interpreted as a state where decision makers can not articulate their preferences clearly due to incomplete information or knowledge, the vagueness of the human thinking, and the inherent complexity and ambiguity of the decision environment.

Group decision making is an important characteristic of modern uncertain decision problem (Levy and Taji, 2007). Organizations often promote the use of “roundtable” meetings in order to facilitate group decision making. The views of one agency may differ from others, but this will

*Corresponding author. E-mail: okyang95@gmail.com. Tel: 886-0922-321006, 886-4-8311498 ext. 3204.

often be to the advantage of the early stages of decision process since it provides a useful forum for all assumptions to be questioned and refined. The end product is thus more realistic. Group decision making handling uncertainty shares have two unique characteristics. First, the group must often make many complex and multi-faceted decisions in a short period of time, thereby contributing to a high “decision load”. Second, the group decision must often be made with incomplete information (both in terms of quantity and quality), whereas, these decisions usually have potentially serious consequences. Maier (1963) uses term “decision quality” to describe the degree to which a wrong decision could lead to catastrophic results. According, the development and application of group DSS could be extremely valuable in the uncertain environment (Ma and Lu, 2010; Wang et al., 2007; Huang et al., 2006).

The Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP) have great potential for use in many practical group decision making problems under uncertainty. For example, Tseng (2010) develops a hybrid MCDM model with the aid of ANP to evaluate firm environment knowledge management in uncertainty. AHP represents a framework with a uni-directional hierarchical AHP relationship (Ding, 2010), while the ANP feedback approach replaces hierarchies with networks in which the relationship between levels are not easily represented as higher or lower, dominator or subordinate, direct or indirect (Meade and Sarkis, 1999). Moreover, ANP is more accurate in complex situation due to its capability of modeling complexity and the way in which comparisons are performed. Hence, the ANP can be considered as a more general form of the AHP in which dependencies and feedbacks between elements of a decision can be modeled.

Although ANP is one of the most complete and comprehensive multi-attribute decision making methods as it encompass the criteria and alternatives in an integrated manner, a great drawback of this method is the pairwise comparison section. This section consists of deterministic comparisons, while it is relatively difficult for decision makers to provide exact numerical values for all the comparison ratios, due to incomplete information or knowledge, complexity and ambiguity within the decision environment, or lack of an appropriate measure units and scale. Therefore, exact numerical values are replaced by Fuzzy judgments for insufficiency and imprecision to incorporate the vagueness of human being in many researches (Razmi et al., 2009; Tuzkaya and önüt, 2008; Tang, 2009). Moreover, many Fuzzy prioritization methods (Huang, 2008; Mikhaiov, 2000, 2003, 2004) are adopted to model the ambiguity and imprecision asso- ciated with the Fuzzy pairwise comparison process, and applied to increase the capabilities of the AHP/ANP (Mikhailov and Singh, 2003; Da deviren and Yüksel, 2010).

In group decision under uncertainty, it is often necessary to combine individual preference to form a group response. There are mainly four basic approaches to estimate group priorities of elements in AHP/ANP (Condon, 2003; Forman and Peniwati, 1998). At first, the decision group is required to reach consensus on every judgments in the matrix. Saaty and Vargas (2007) note that to achieve a decision with which the group is satisfied, it is necessary for the judgments to be homogeneous. If consensus is not possible, the second approach is to use a vote on the various judgments proposed to pick a compromise for the value of the group entry. A stream of researches has used these basic techniques for deriving group priorities in ANP (Liu and Lai, 2009; Nekhy et al, 2009; Boran and Goztepe, 2010). If the group is unwilling or unable to vote or can achieve a consensus under uncertainty, then a third group solution can be obtained by aggregating the judgments of the decision makers for each set of pairwise comparisons into a new set of aggregated group judgments at each level of comparisons.

The aggregated group judgments are considered as judgments of a ‘new individual’ and the priorities of this individual are derived as a group solution. However, applying this method requires a full set of comparison judgments. In the practical group decision under uncertainty, it is unlikely for all decision makers to give all the comparison judgments, and then the application of this method is problematical, since usually the group members have different level of expertise. A fourth group aggregation approach is aggregating individual priorities into group preferences. Individual priority vectors from comparison judgment matrices can be easily derived by Fuzzy prioritization methods.

Nevertheless, the subsequent aggregation process become intractable, for some of the resulted individual, priority vectors are incomplete when the exact individuals are unwilling or unable to evaluate all the elements under uncertainty. Therefore, there are challenges associated with each of these four approaches, and an appropriate group prioritization method should be developed to extend ANP to deal with uncertain decision making problems.

Hence, Fuzzy sets theory and group ANP method are integrated to investigate a method for designing a group DSS under uncertainty. The group DSS under uncertainty formulated in this paper comprises of the following steps.

First, ANP instead of AHP is proposed due to the fact that ANP can accommodate the variety of interactions, dependencies and feedback between higher and lower level elements. Second, Fuzzy judgments are introduced in the pairwise comparison of ANP to make up the deficiency in order to capture the right judgments of decision makers in the conventional ANP.

Third, a Fuzzy prioritization method is proposed to derive the local priorities from uncertain pairwise comparison judgments. Fourth, the elicited local priorities are further aggregated into group priorities by an original aggregation method, which can cope with the situations where some of the local priorities are missing when the decision makers do not evaluate some of elements under uncertainty.

PROBLEM OF GROUP DECISION UNDER UNCERTAINTY MODELING

Fuzzy judgments have been widely used in ANP to express the subjective uncertainty in preference. Consider a group of N decision makers DM_k $(k = 1,2,\ldots,N)$ evaluate n elements (clusters, criteria or alternatives) E_i $(i = 1,2,\ldots,n)$. Each decision maker provides a set $\Theta_k = \{\tilde{a}_{ijk}\}$ of $m \le n(n-1)/2$ fuzzy comparison judgments, $i = 1,2,\ldots,n-1$, $j = 2,3,\ldots,n$, $j > i$, $k = 1,2,\ldots,N$, where $\tilde{a}_{ijk}$ represents the relative importance of the decision element E_i over E_j, assessed by the kth decision maker.

Triangular fuzzy numbers $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$ are used to represent the fuzzy judgments, where l_{ijk}, m_{ijk} and u_{ijk} are the lower, modal and upper bounds respectively, satisfying a reciprocity condition analogous to that of standard pairwise comparison matrices. When these judgments are consistent, there are many local priority vectors, whose elements ratios satisfy the inequalities.

$$l_{ijk} \le \frac{\omega_i^k}{\omega_j^k} \le u_{ijk} , \qquad (1)$$

In inconsistent cases, however, a priority vector that satisfies all inequalities (1) simultaneously, does not exist. But it is reasonable to yields a crisp local priority vector $\omega_k = (\omega_{1k}, \omega_{2k}, \ldots, \omega_{nk})^T$, such that the priority ratios ω_{ik}/ω_{kj} approximately satisfy the initial fuzzy judgments.

$$l_{ijk} \tilde{\le} \frac{\omega_i^k}{\omega_j^k} \tilde{\le} u_{ijk} , \qquad (2)$$

where the symbol $\tilde{\le}$ denotes 'approximately less or equal to'.

However, in most uncertain decision problem, it is unlikely for all decision-makers to provide all the comparisons among elements or evaluate all elements due to incomplete information or knowledge, the vagueness of human thinking, and the inherent complexity and uncertaintainty of decision environment. Let ω_{ik} denote the local priority of decision maker DM_k evaluating element E_i. Note that $\omega_i^k > 0$ holds whenever DM_k evaluate E_i and otherwise $\omega_{ik} = 0$. Consider a bipartite graph with two node sets $K = \{1,2,\ldots,N\}$ and $I = \{1,2,\ldots,n\}$, corresponding to decision makers and elements, respectively, and the arc (DM_k, E_i), where $k \in K$ and $i \in I$, exists if only DM_k evaluate E_i. In other words, the arc (DM_k, E_i) exists if and only if $\omega_{ik} > 0$.

To clarify, consider the case of four decision- makers and three elements, as shown in Figure 1.

The bipartite graph of group decision under uncertainty, as showed in Figure 1, illustrates the group decision's evaluation situations using Fuzzy judgments under uncertainty as follows:

(i) Decision maker DM_k evaluates all elements by providing a full set of pairwise comparison judgments, and all the arcs are solid. For example, DM_1 and DM_3 make all the pairwise comparisons judgments of all elements, and it follows that all the local priorities are positive.
(ii) Decision maker DM_k evaluates all elements by comparing some pairs of elements and neglecting other pairs. For example, DM_2 provides the relative importance of the element E_1 over E_2, and E_1 over E_3, and does not compare the elements E_2 with E_3 directly. The relationship between DM_2 and E_1 is denoted by solid arc, and the arcs (DM_2, E_2) and (DM_2, E_3) are can be denoted by dotted ones. While DM_2 makes fuzzy pairwise comparison judgments with missing information and all the local priorities are positive.
(iii) Decision maker DM_k evaluates some of the elements, but neglects other elements. For example, DM_4 only estimates the relative importance of the element E_1 over E_3, without any comparisons about the element E_2. Obviously DM_4 provides a Fuzzy pairwise comparison matrix without any preference information about the element E_2, it follows that the local priorities ω_{14} and ω_{34} are positive and the other local priority ω_{24} is zero.
(iv) Decision maker DM_k performs inconsistent judgments.

The Fuzzy prioritization method developed by Mikhailow is proposed to deals with the aforementioned situations of Fuzzy judgments (in (ii), (iii) and (iv)), where there are missing or inconsistent judgments in the Fuzzy pairwise comparison matrices for its advantage of measuring consistency indexes for the Fuzzy pairwise comparison matrixes.

After the local priorities are derived, the combination of individual priorities into group priorities is considered. An original aggregation method that uses nonlinear programming is developed to cope with the situation in (iii) where there are zero local priorities.

GROUP FUZZY ANP UNDER UNCERTAINTY

Fuzzy prioritization method

The Fuzzy prioritization approach formulates the derivation of priorities from Fuzzy judgments as an optimi-zation problem that maximizes the "the decision-maker's overall satisfaction with the final solution. Membership

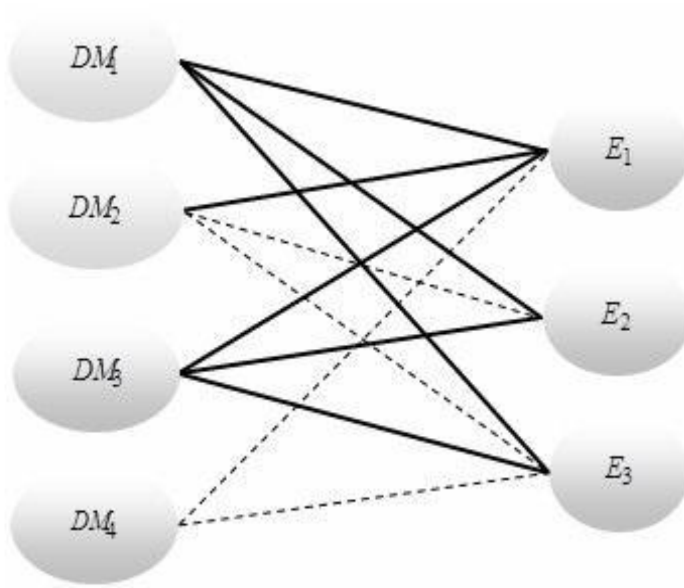

Figure 1. Bipartite graph of group decision under uncertainty.

functions that represent the decision-maker's satisfaction with different crisp solution ratios ω_{ik}/ω_{kj} could be introduced. Each crisp priority vector ω^k satisfies the double-side inequality (2) with some degree, which can be measured by a membership function, linear with respect to the unknown ratio ω_{ik}/ω_{kj}.

$$\mu_{ij}^{L}\left(\frac{\omega_i^k}{\omega_j^k}\right)=\begin{cases}\dfrac{(\omega_i^k/\omega_j^k)-l_{ijk}}{m_{ijk}-l_{ijk}}, & \omega_i^k/\omega_j^k \le m_{ijk},\\[2ex] \dfrac{u_{ijk}-(\omega_i^k/\omega_j^k)}{u_{ijk}-m_{ijk}}, & \omega_i^k/\omega_j^k \ge m_{ijk}.\end{cases} \quad (3)$$

The linear membership function (Equation (3)) represents an L-Fuzzy set $L=(-\infty,1]$, which is not bound from above and blew, and the shape of it is shown in Figure 2. It is seen that this membership function is linearly increasing over the interval $(-\infty, m_{ijk}]$, and linearly decreasing over the interval $[m_{ijk}, \infty)$. The membership function has a maximum value $\mu_{\max L}=1$ at $\omega_{ik}/\omega_{kj}=m_{ijk}$. The degree of membership $\mu_{ijL}(\omega_k)$ is positive over the range $[l_{ijk}, u_{ijk}]$, when the decision maker is satisfied, the corresponding priority vector ω_k, and $\mu_{ijL}(\omega_k)$ is negative when $\omega_i^k/\omega_j^k < l_{ijk}$ or $\omega_i^k/\omega_j^k > u_{ijk}$, which indicates dissatisfaction with the solution.

The solution to the prioritization problem by Fuzzy prioritization method is based on two main assumptions. The first one requires the existence of non-empty Fuzzy feasible area p_k on the $(n-1)$-dimensional simplex Q^{n-1} plane:

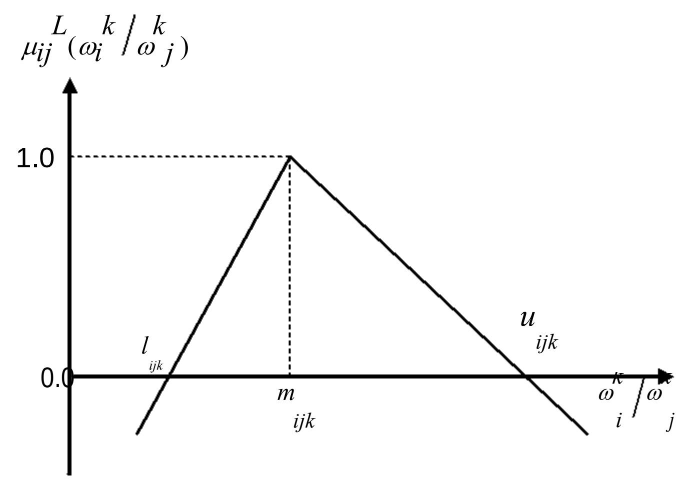

Figure 2. Member function, linear in the ratio space.

$$Q^{n-1}=\left\{(\omega_1^k,\omega_2^k,\ldots,\omega_n^k)\,\middle|\,\omega_i^k>0,\ \sum_{i=1}^{n}\omega_i^k=1\right\} \quad (4)$$

Defined as an intersection of the membership functions, similar to Equation (3) and the simplex hyperplane in Equation (4), the membership function of the Fuzzy feasible area p_k is given by:

$$\mu_{p_k}(\omega^k)=\min_{ij}\left\{\mu_{ij}(\omega_i^k,\omega_j^k)\,\middle|\,i=1,2,\ldots,n;\ j=2,3,\ldots,n;\ j>j\right\}. \quad (5)$$

By defining the membership functions in Equation (3) as L-Fuzzy sets $\{L=(-\infty,1]\}$, the assumption of non-emptiness of p_k on the simplex could be relaxed. If the Fuzzy judgments are very inconsistent, then $\mu_{p^k}(\omega_k)$ could take negative values for all normalized priority vectors $\omega_k \in Q_{n-1}^{-}$.

The second assumption of the Fuzzy prioritization method specifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function as seen in Equation (5). It can easily be proved $\mu_{p^k}(\omega_k)$ is a convex set, so there is always a priority vector $\omega_{*k} \in Q_{n-1}$ that has a maximum degree of membership:

$$\lambda_*^k = \mu_{p^k}(\omega_*^k) = \max_{\omega^k \in Q^{n-1}} \min_{ij} \left\{ \mu_{ij}(\omega^k) \right\} \qquad (6)$$

The maximum prioritization problem (6) can be represented in the following way:

$$\begin{aligned} &\max imise \quad \lambda^k \\ &subject\ to \quad \lambda_k \le \mu_{ij}(\omega^k), \\ &\qquad i = 1,2,\ldots,n-1,\ j = 2,3,\ldots,n\ \ j > i, \\ &\qquad \sum_{l=1}^{n} \omega_l^k = 1,\ \ \omega_l^k > 0,\qquad l = 1,2,\ldots,n. \end{aligned} \qquad (7)$$

Taking into consideration the specific form of the membership functions (Equation 3), the problem (Equation 7) can be further transformed into a bilinear program of the type:

$$\begin{aligned} &\max imise \quad \lambda^k \\ &subject\ to \quad (m_{ijk} - l_{ijk})\lambda^k \omega_j^k - \omega_i^k + l_{ijk}\omega_j^k \le 0, \\ &\qquad (u_{ijk} - m_{ijk})\lambda^k \omega_j^k + \omega_i^k - u_{ijk}\omega_j^k \le 0, \\ &\qquad i = 1,2,\ldots,n-1,\quad j = 2,3,\ldots,n,\quad j > i, \\ &\qquad \sum_{l=1}^{n} \omega_l^k = 1,\ \ \omega_l^k > 0,\ \ l = 1,2,\ldots,n. \end{aligned} \qquad (8)$$

The optimal value λ_*^k, if it is positive, indicates that all solution ratios completely satisfy the Fuzzy judgments. For example, $l_{ijk} \le (\omega_{i*}^k / \omega_{j*}^k) \le u_{ijk}$, which means that the initial set of Fuzzy judgments is rather consistent. A negative value of λ_{*k} shows that the solutions ratios that approximately satisfy all the double-side inequalities (Equation 3). For example, the Fuzzy judgments are strongly inconsistent. Therefore, the optimal value λ_{*k} can be used for measuring the consistency of the initial set of Fuzzy judgment.

Nonlinear programming methods to group aggregation under uncertainty

In order to aggregate individual priorities into group priorities, compromise is necessary. Therefore, the main objective of the decision group can be transformed to generate a compromise solution that minimizes the inconformity existing between individual priorities and group priorities. Nonlinear programming methods can be applied to minimize the inconformity to maximize the unanimous group priorities. As shown in section 2, in the uncertain decision environment, once the decision maker DM_k evaluates element E_i, the arc (DM_k, E_i) exists, and the local priority $\omega_i^k > 0$, otherwise, the arc (DM_k, E_i) does not exist, and the local priority $\omega_{ik} = 0$. Based on these observations, an aggregation method is proposed by which the group priority vector for elements $\omega_g = (\omega_{g1}, \omega_{g2}, \ldots, \omega_{gn})^T$ is obtained as the solution ω_g^* to the following nonlinear programming problem:

$$\begin{aligned} &\min imise \quad Q(\omega_g) = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{N} \left(\omega_i^k - H(\omega_i^k)\omega_{gi}\right)^2, \\ &subject\ to \quad \sum_{i=1}^{n} \omega_{gi} = 1,\ i = 1,2,\ldots,n, \end{aligned} \qquad (9)$$

with $H(\cdot)$ being a function defined as:

$$H(t) = \begin{cases} 1 & t > 0 \\ 0 & t \le 0 \end{cases}$$

Theorem 1

When the decision maker DM_k evaluates element E_i, for all i, the nonlinear programming problem (9) has a global optimal solution $\omega_g^* = \left(\omega_{g1}^*, \omega_{g2}^*, \ldots, \omega_{gn}^*\right)^T$, and

$$\omega_{gi}^* = \frac{1 + b_i \sum_{i=1}^{n} 1/a_i - \sum_{i=1}^{n} b_i/a_i}{a_i \sum_{i=1}^{n} 1/a_i},\quad i = 1,2,\ldots,n, \qquad (10)$$

where $a_i = \sum_{k=1}^{N} H^2(\omega_i^k)$, and $b_i = \sum_{k=1}^{N} H(\omega_i^k)\omega_i^k$.

Proof

See the appendix A.

Steps of group Fuzzy ANP model under uncertainty

The process of applying the group Fuzzy ANP under uncertainty that combines Fuzzy prioritization method, nonlinear programming for group decision and ANP comprises of the following main steps:

Step 1: Identify alternatives, criteria and clusters to be used in the proposed model.

Step 2: Configure a network structure including clusters, criteria, alternatives and dependences among these

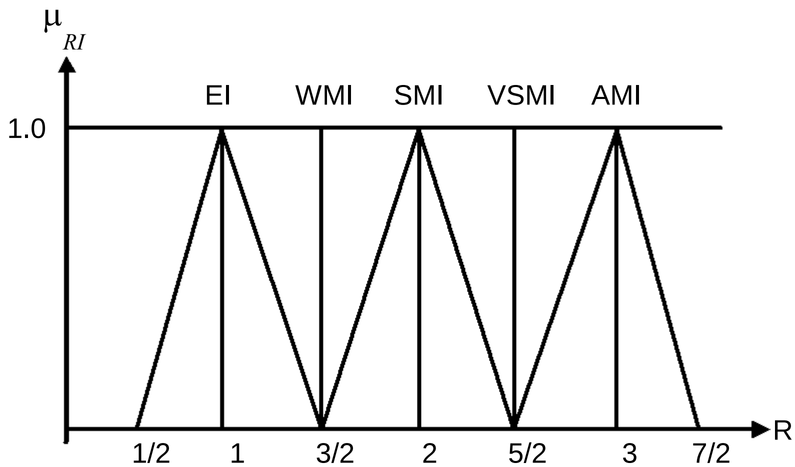

Figure 3. Linguistic scale for relative importance.

Table 1. Linguistic scale for relative importance.

Linguistic scale for importance	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Equally important (EI)	$(1/2, 1, 3/2)$	$(2/3, 1, 2)$
Weakly more important (WMI)	$(1, 3/2, 2)$	$(1/2, 2/3, 1)$
Strongly more important (SMI)	$(3/2, 2, 5/2)$	$(2/5, 2, 2/3)$
Very strongly more important (VSMI)	$(2, 5/2, 3)$	$(1/3, 2/5, 1/2)$
Absolutely more important (AMI)	$(5/2, 3, 7/2)$	$(2/7, 1/3, 2/5)$

components.

Step 3: Construct pairwise matrices of the components with Fuzzy judgments. The Fuzzy scale regarding relative importance to measure the relative priorities is given in Figure 3 and Table 1. Similarly scale is proposed by Kahraman et al. (2006) and used for solving Fuzzy decision-making problems (Kahraman et al., 2006; Razmi et al., 2009) in the literature. This scale will be used in the Fuzzy prioritization approach.

Step 4: Determine the local priorities and consistency index from each matrix using the fuzzy prioritization method.

Step 5: Check the consistency index. If it is fairly accepted, continue, otherwise return to step 3.

Step 6: Aggregate local priorities into group priorities using nonlinear programming approach as explained.

Step 7: Fill the super matrix with the elicited group priorities to form unweighted supermatrix.

Step 8: Obtain weighted supermatrix by multiplying the unweighted supermatrix by the corresponding cluster priorities, and then adjusting the resulting supermatrix to column stochastic.

Step 9: Limit the weighted supermatrix by raising it to sufficiently large power so that it converges into a stable supermatrix (all columns being identical).

Step 10: Normalize the scores of alternatives from the limit weighted supermatrix into final priorities.

Petroleum contaminated site remedial countermeasures example

In this section, an evaluation of petroleum contaminated site remedial countermeasures is considered to demonstrate how the proposed model is applied. A Chinese petroleum enterprise intended to select the most appropriate remedial alternatives for a contaminated site caused by a petroleum pipeline leak. In this decision contest, four key decision makers DM_k $(k = 1, 2, , 4)$ are authorized to prioritize the feasible remedial alternatives and select the best countermeasure for the site. Five remedial countermeasure strategies are briefly described in Table 2. Similar decision problem of choosing the best contaminated site remedial countermeasure is given in Promentilla (2006) study's. The evaluation criteria to choose the best remedial alternative for the contaminated site are defined as followed: C_1 , the social acceptability of the countermeasures according to the perception of stakeholders; C_2 , implementability in terms of administrative and technological feasibility ; C_3 , financial affordability with regards to the overall cost of the clean-up; C_4 , environmental effectiveness to protect public health and environment resources . In the next step, criteria are grounded into two clusters: External environment (including C_1 and C_4), and internal capabilities (including

Table 2. The remedial countermeasures in this illustrative example.

Remedial alternatives	Site remediation containing the waste layer	Prevention of contaminant spreading	Remediation of surrounding area
Alternative 1 (A_1)	*In situ* disposal by incineration is a commercial possibility, if the volume of the petroleum-contaminated soil is large enough		*In situ* remediation (e.g. enhanced bioremediation, natural attenuation)
Alternative 2 (A_2)	Complete removal of waste from the petroleum-contaminated soil from the site, and off-site treatment and disposal of the excavated waste		*In situ* remediation (e.g. enhanced bioremediation, natural attenuation)
Alternative 3 (A_3)	*In situ* remediation (e.g., enhanced bioremediation, soil washing, etc.)	Capping and plume control (e.g., groundwater extraction)	*In situ* remediation (e.g. enhanced bioremediation, natural attenuation)
Alternative 4 (A_4)	*In situ* remediation (e.g., enhanced bioremediation, soil washing, etc.)	Capping and vertical cut-off wall (e.g., sheet piling, chemical grout, etc.)	In situ remediation (e.g. enhanced bioremediation, natural attenuation)
Alternative 5 (A_5)	*In situ* remediation (e.g. enhanced bioremediation, natural attenuation)		

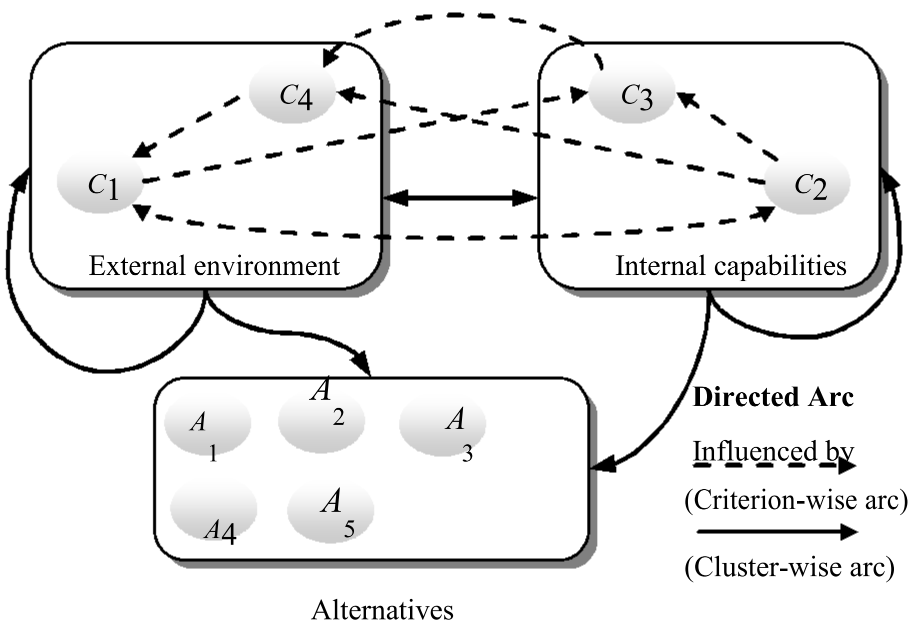

Figure 4. Network of petroleum contaminated site remedial countermeasures selection.

C_2 and C_3). The dependencies between criteria are as follows:

1. Social acceptability, financial affordability and environmental effectiveness depends on implementability;
2 .Implementability and financial affordability are directly affected by social acceptability;
3. Financial affordability influence environmental effectiveness; and
4. Social acceptability depends on environmental effectiveness.

Moreover, it must be noted that all alternatives are influenced by the four aforementioned defined criteria. Regarding this relation, the network of the problem is formed as shown in Figure 4. The solid arrows represents the dependencies within the clusters resulted from criteria dependencies within the clusters, and the dotted arrows show connections between criteria within one cluster or two different clusters.

The following pairwise comparison matrices are performed with respect to the aforementioned network and are formed by the four key decision makers by using the

Table 3. Local priorities and pairwise comparison matrix of alternatives with respect to social acceptability.

Social acceptability	A_1	A_2	A_3	A_4	A_5	**Local priorities**
DM_1						$\lambda^{1}_{*} = 0.629$
A_1	(1,1,1)	(1/2,2/3,1)			(2/3,1,2)	0.186
A_2	(1,3/2,2)	(1,1,1)	(1/2,1,3/2)	(1,3/2,2)		0.235
A_3		(2/3,1,2)	(1,1,1)	(1/2,1,3/2)	(1/2,1,3/2)	0.199
A_4		(1/2,2/3,1)	(2/3,1,2)	(1,1,1)	(1/2,2/3,1)	0.168
A_5	(1/2,1,3/2)		(2/3,1,2)	(1,3/2,2)	(1,1,1)	0.212
DM_2						$\lambda^{2}_{*} = 0.653$
A_1	(1,1,1)	(2/3,1,2)	(2/3,1,2)	(1/2,2/3,1)		0.182
A_2	(1/2,1,3/2)	(1,1,1)	(1/2,1,3/2)	(2/3,1,2)	(1/2,1,3/2)	0.205
A_3	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)	(2/3,1,2)	(2/3,1,2)	0.206
A_4	(1,3/2,2)	(1/2,1,3/2)	(1/2,1,3/2)	(1,1,1)	(1,3/2,2)	0.232
A_5		(2/3,1,2)	(1/2,1,3/2)	(1/2,2/3,1)	(1,1,1)	0.175
DM_3						$\lambda^{3}_{*} = 0.745$
A_1						
A_2		(1,1,1)	(1,3/2,2)	(1,3/2,2)	(3/2,2,5/2)	0.342
A_3		(1/2,2/3,1)	(1,1,1)	(2/3,1,2)	(2/3,1,2)	0.228
A_4		(1/2,2/3,1)	(1/2,1,3/2)	(1,1,1)	(1,3/2,2)	0.249
A_5		(2/5,1/2,2/3)	(1/2,1,3/2)	(1/2,2/3,1)	(1,1,1)	0.181
DM_4						$\lambda^{4}_{*} = 0.753$
A_1						
A_2		(1,1,1)	(2,5/2,3)	(2/3,1,2)		0.405
A_3		(1/3,2/5,1/2)	(1,1,1)	(2/7,1/3,2/5)		0.154
A_4		(1/2,1,3/2)	(5/2,3,7/2)	(1,1,1)		0.441
A_5						

scale given in Figure 3 and Table 2. All components of the alternatives cluster are compared, pairwisely, with respect to each of the criteria. Implementability and financial affordability are compared with respect to social acceptability. In addition, social acceptability and environmental effectiveness are compared with respect to implementability. With respect to external environment; alternatives, internal capabilities and external environment clusters are compared, pairwisely. Finally, alternatives, external environment, and internal capabilities clusters are compared with respect to internal capabilities. The Fuzzy prioritization approach, as explained, is used for calculating local priorities, and the nonlinear programming method, as introduced, is applied to aggregate the local priorities into group priorities. The comparison matrices of the alternatives with respect to social acceptability are demonstrated in Table 3. The local priorities from the comparison matrix performed by DM_1 are calculated by solving the following model whit Lingo V12.0 software:

$\max = \lambda$

subject to

$(1/6)\times\lambda\times\omega 2-\omega 1+(1/2)\times\omega 2<=0$;

$(1/3)\times\lambda\times\omega 2+\omega 1-\omega 2<=0$;

$(1/3)\times\lambda\times\omega 5-\omega 1+(2/3)\times\omega 5<=0$;

$\lambda\times\omega 5+\omega 1-2\times\omega 5<=0$;

$(1/2)\times\lambda\times\omega 3-\omega 2+(1/2)\times\omega 3<=0$;

$(1/2)\times\lambda\times\omega 3+\omega 2-(3/2)\times\omega 3<=0$;

$(1/2)\times\lambda\times\omega 4-\omega 2+\omega 4<=0$;

$(1/2)\times\lambda\times\omega 4+\omega 2-2\times\omega 4<=0$;

$(1/2)\times\lambda\times\omega 4-\omega 3+(1/2)\times\omega 4<=0$;

$(1/2)\times\lambda\times\omega 4+\omega 3-(3/2)\times\omega 4<=0$;

$(1/2)\times\lambda\times\omega 5-\omega 3+(1/2)\times\omega 5<=0$;

$(1/2)\times\lambda\times\omega 5+\omega 3-(3/2)\times\omega 5<=0$;

$(1/6)\times\lambda\times\omega 5-\omega 4+(1/2)\times\omega 5<=0$;

$(1/3)\times\lambda\times\omega 5+\omega 4-\omega 5<=0$;

$\omega 1+\omega 2+\omega 3+\omega 4+\omega 5=1$;

end

Table 4. Local priorities under five criteria for alternatives.

Social acceptability	DM_1	DM_2	DM_3	DM_4	Implementability	DM_1	DM_2	DM_3	DM_4
A_1	0.186	0.182			A_1	0.258		0.354	0.491
A_2	0.235	0.205	0.342	0.405	A_2	0.123			0.165
A_3	0.199	0.206	0.228	0.154	A_3			0.250	
A_4	0.168	0.232	0.249	0.441	A_4	0.362	0.550	0.396	0.344
A_5	0.212	0.175	0.181		A_5	0.257	0.450		
Financial affordability	DM_1	DM_2	DM_3	DM_4	**Environmental effectiveness**	DM_1	DM_2	DM_3	DM_4
A_1	0.173	0.229		0.305	A_1	0.100		0.220	0.023
A_2	0.102	0.200	0.165	0.168	A_2	0.267	0.354	0.250	0.322
A_3	0.252	0.203	0.344	0.371	A_3	0.229	0.276		0.311
A_4	0.242	0.177	0.491	0.156	A_4	0.268	0.310	0.278	0.223
A_5	0.231	0.191			A_5	0.136	0.060	0.252	0.121

Table 5. Group priorities under five criteria for five alternatives.

	A_1	A_2	A_3	A_4	A_5	**Ranking**
Social acceptability	0.140	0.275	0.175	0.250	0.160	A_2 A_4 A_3 A_5 A_1
Implementability	0.299	0.042	0.046	0.326	0.251	A_4 A_1 A_5 A_3 A_2
Financial affordability	0.201	0.133	0.267	0.240	0.159	A_3 A_4 A_1 A_5 A_2
Environmental effectiveness	0.092	0.281	0.249	0.253	0.125	A_2 A_4 A_3 A_5 A_1

Thus, the local priority vector from the aforementioned model is calculated as $\omega_1 = (0.186, 0.235, 0.199, 0.168, 0.212)^T$. Consistency index (λ_{1*}) is calculated as 0.629 and this rate suggests that the Fuzzy comparison matrix is consistent. Pairwise comparison matrices performed by DM_2, DM_3 and DM_4 for the alternatives with respect to social acceptability are given in Table 3 together with the calculated local priorities and consistency index. Table 4 illustrates the local priorities provided by the four key decision makers DM_k $(k = 1, 2, , 4)$ for the alternatives with respect to all four criteria.

Since the decision making group has evaluated all the five alternatives with respect to each criterion as shown in Table 4, the group priorities of them are uniquely determined (according to Theorem 1). Table 5 shows the group priorities under each criterion aggregated by the optimization problem in Equation (9). According to the ranking of all alternatives under different criterion, there is no dominating alternative for all the criteria. For example, A_2 is perceived to dominate in terms of environmental effectiveness and social acceptability, but its ratings are the lowest in terms of financial affordability and imple-mentability because of its potentially high clean- up cost, as well as, difficulties in treatment facility for the excavated waste. On the hand, A_3 has the least clean-up cost, but it may also have some difficulties in the effectively control of the plume. A_1 has good performances in terms of financial affordability and implementability. However, its desirability is the lowest in terms of environmental effectiveness and social acceptability. In the case of A_4, it is perceived to dominate in terms of implementability, but its desirability is relatively lower with respect to the other criteria in which A_2 and A_3 dominates. In order to further prioritize these five alternatives for remediation design and analysis, it is necessary to combine the pairwise comparisons of criteria and clusters to evaluate these alternatives and measure their overall relative desirability.

Comparison matrices of the internal capability cluster with respect to social acceptability, and the external environment cluster with respect to implementability, as shown in Tables 6 - 7, are performed by the four key decision makers by reaching consensus on every entry, since there is only one pair of Fuzzy judgment in each comparison matrix. The priorities listed in Tables 6 – 7, are also calculated by the Fuzzy prioritization method and the consistency indices are found to be one. For cluster comparison, the resulted matrices together with their local priorities, consistency indices and group priorities are shown in Tables 8 – 9.

The unweighted supermatrix is formed by putting the group priorities in the corresponding block of the matrix

Table 6. Group priorities and comparison matrix of the internal capabilities cluster with respect to social acceptability.

Social acceptability	Implementability	Financial affordability	Group priorities
implementability	(1,1,1)	(2,5/2,3)	0.714
financial affordability	(1/3,2/5,1/2)	(1,1,1)	0.286

Table 7. Group priorities and comparison matrix of the external environment cluster with respect to implementability.

Implementability	Social acceptability	Environmental effectiveness	Group priorities
Social acceptability	(1,1,1)	(1/2,1,3/2)	0.500
Environmental effectiveness	(2/3,1,2)	(1,1,1)	0.500

Table 8. Priorities and comparison matrix of the clusters with respect to external environment.

External environment	Alternatives	Internal capabilities	External environment	Local priorities
DM_1				$\lambda^1_* = 0.333$
Alternatives	(1,1,1)	(1/3,2/5,1/2)	(2/3,1,2)	0.226
Internal capabilities	(2,5/2,3)	(1,1,1)	(2/3,1,2)	0.484
External environment	(1/2,1,3/2)	(1/2,1,3/2)	(1,1,1)	0.290
DM_2				$\lambda^2_* = 1.000$
Alternatives	(1,1,1)		(1/2,1,3/2)	0.334
Internal capabilities		(1,1,1)	(2/3,1,2)	0.333
External environment			(1,1,1)	0.333
DM_3				$\lambda^3_* = 0.802$
Alternatives	(1,1,1)	(2/7,1/3, 2/5)	(1/2,1,3/2)	0.200
Internal capabilities		(1,1,1)	(2,5/2,3)	0.578
External environment			(1,1,1)	0.222
DM_4				$\lambda^4_* = 1.000$
Alternatives	(1,1,1)	(1/3, 2/5,1/2)		0.222
Internal capabilities		(1,1,1)	(2,5/2,3)	0.556
External environment			(1,1,1)	0.222
Group priorities	0.245	0.488	0.267	

(Table 10). Then, the weighted supermatrix is obtained by multiplying unweighted supermatrix by their corres-ponding group priorities of clusters (Tables 8 - 9), and then adjusted to be column stochastic. Consequently, the limit supermatrix is the stable powered form of that of the weighted. Tables 11 and 12 demonstrate the weighted and the limit supermatrices, respectively. It must be noted that the first five columns of the relative supermatrices are zero, because the decision criteria does not depend on the alternatives as can be seen in Figure 3.

In Table 12, values in rows in front of the alternatives are the final scores of the alternatives, and can be normalized into final priorities, as shown in Table 8. Considering the final priorities $\omega_{final} = (0.184,0.177,0.193.0.274,0.172)$ of all alternatives, their ranking is as follows: A_4 A_3 A_1 A_2 A_5 . Based on this analysis, A_4 is identified as the most preferred

Table 9. Priorities and comparison matrix of clusters with respect to internal capabilities.

Internal capabilities	Alternatives	Internal capabilities	External environment	local priorities
DM_1				$\lambda^1_* = 0.708$
Alternatives	(1,1,1)	(1/2,2/3,1)	(1/2,1,3/2)	0.282
Internal capabilities	(1,3/2,2)	(1,1,1)	(3/2,2,5/2)	0.464
External environment	(2/3,1,2)	(2/5,1/2,2/3)	(1,1,1)	0.254
DM_2				$\lambda^2_* = 1.000$
Alternatives	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.500
Internal capabilities	(2/5,1/2,2/3)	(1,1,1)	(1/2,1,3/2)	0.250
External environment	(2/5,1/2,2/3)	(2/3,1,2)	(1,1,1)	0.250
DM_3				$\lambda^3_* = 1.000$
Alternatives	(1,1,1)		(1/2,1,3/2)	0.334
Internal capabilities		(1,1,1)	(2/3,1,2)	0.333
External environment	(2/3,1,2)	(1/2,1,3/2)	(1,1,1)	0.333
DM_4				$\lambda^4_* = 0.796$
Alternatives	(1,1,1)	(5/2,3,7/2)	(2,5/2,3)	0.578
Internal capabilities	(2/7,1/3, 2/5)	(1,1,1)	(1/2,1,3/2)	0.200
External environment	(1/3, 2/5,1/2)	(2/3,1,2)	(1,1,1)	0.222
Group priorities	0.424	0.312	0.264	

Table 13. Final scores and ranking of the alternatives.

Alternatives	Final scores (Normal values)	Ranking
A_1	0.184	3
A_2	0.177	4
A_3	0.193	2
A_4	0.274	1
A_5	0.172	5

alternative, although A_2 is the most preferred alternative according to environmental effectiveness and social acceptability, and A_3 is the most favorable alternative with respect to cost reduction. The Top performance of A_4 isdue to its excellent implementability in terms of administrative and technological feasibility, good cost reduction, appropriate effectiveness to protect public health and environment resources, and medium social acceptability.

Conclusions

A group DSS under uncertainty using group Fuzzy ANP approach has been systematically developed and applied in the evaluation of petroleum contaminated site remedial countermeasures. In order to handle uncertainty, a bipartite graph is firstly formulated to model the problem of group decision making under uncertainty. Then, a group Fuzzy ANP approach combining Fuzzy prioritization method, nonlinear programming and ANP is designed and applied to the group DSS for ranking the alternatives while taking uncertainty into account. Finally, the designed group DSS is used to evaluate petroleum contaminated site remedial countermeasures for an oil company in China, and the proposed group DSS demonstrates great effectiveness in handling uncertainty and supporting group decision making with high level of user satisfaction.

The formulated group Fuzzy ANP approach for group DSS has the following improvements compared with the conventional ANP:

1. Due to partially known information, fuzzy judgments are more adaptive to characterize the uncertainty compared with point-valued judgments in the conventional ANP.
2. A Fuzzy modification of the ANP is proposed to derive local priorities form uncertain Fuzzy pairwise comparison judgment.
3. An original aggregation method is integrated into the group DSS to cope with the situation where some of the local priorities are missing for the likeliness in that decision makers do not evaluate some of the elements under uncertainty.
4. Decision makers can make judgments individually, their

Table 10. Unweighted supermartrix to select the best petroleum contaminated site remedial countermeasure.

	A_1	A_2	A_3	A_4	A_5	**Environmental effectiveness**	**Social acceptability**	**Financial affordability**	**Implementability**
A_1	0.000	0.000	0.000	0.000	0.000	0.092	0.140	0.201	0.299
A_2	0.000	0.000	0.000	0.000	0.000	0.281	0.275	0.133	0.042
A_3	0.000	0.000	0.000	0.000	0.000	0.249	0.175	0.267	0.046
A_4	0.000	0.000	0.000	0.000	0.000	0.253	0.250	0.240	0.362
A_5	0.000	0.000	0.000	0.000	0.000	0.125	0.160	0.159	0.251
Environmental effectiveness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.500
Social acceptability	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.500
Financial affordability	0.000	0.000	0.000	0.000	0.000	0.000	0.286	0.000	1.000
Implementability	0.000	0.000	0.000	0.000	0.000	0.000	0.714	0.000	0.000

Table 11. Weighted supermartrix to select the best petroleum contaminated site remedial countermeasure.

	A_1	A_2	A_3	A_4	A_5	**Environmental effectiveness**	**Social acceptability**	**Financial affordability**	**Implementability**
A_1	0.000	0.000	0.000	0.000	0.000	0.044	0.047	0.124	0.127
A_2	0.000	0.000	0.000	0.000	0.000	0.134	0.092	0.082	0.018
A_3	0.000	0.000	0.000	0.000	0.000	0.119	0.058	0.165	0.020
A_4	0.000	0.000	0.000	0.000	0.000	0.121	0.084	0.148	0.153
A_5	0.000	0.000	0.000	0.000	0.000	0.060	0.053	0.098	0.106
Environmental effectiveness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.384	0.312
Social acceptability	0.000	0.000	0.000	0.000	0.000	0.522	0.000	0.000	0.312
Financial affordability	0.000	0.000	0.000	0.000	0.000	0.000	0.190	0.000	0.312
Implementability	0.000	0.000	0.000	0.000	0.000	0.000	0.475	0.000	0.000

thoughtfulness and responsibilities are fully demonstrated in the group DSS, which improves group decision making transparency.

5. The group fuzzy ANP approach is more preferred in dealing with the uncertainty. Future research goals include extending the group Fuzzy ANP approach to larger and more complex network structure, carrying out sensitivity analysis, and figuring out the contribution value of every decision maker for the final judgment.

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Table 12. Limit supermartrix to select the best petroleum contaminated site remedial countermeasure.

	A_1	A_2	A_3	A_4	A_5	Environmental effectiveness	Social acceptability	Financial affordability	Implementability
A_1	0.000	0.000	0.000	0.000	0.000	0.084	0.084	0.084	0.084
A_2	0.000	0.000	0.000	0.000	0.000	0.081	0.081	0.081	0.081
A_3	0.000	0.000	0.000	0.000	0.000	0.088	0.088	0.088	0.088
A_4	0.000	0.000	0.000	0.000	0.000	0.125	0.125	0.125	0.125
A_5	0.000	0.000	0.000	0.000	0.000	0.079	0.079	0.079	0.079
Environmental effectiveness	0.000	0.000	0.000	0.000	0.000	0.125	0.125	0.125	0.125
Social acceptability	0.000	0.000	0.000	0.000	0.000	0.153	0.153	0.153	0.153
Financial affordability	0.000	0.000	0.000	0.000	0.000	0.131	0.131	0.131	0.131
Implementability	0.000	0.000	0.000	0.000	0.000	0.134	0.134	0.134	0.134

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$i=1$

APPENDIX A

Theorem 1

When the decision maker DM_k evaluates element E_i , for all i , the nonlinear programming problem (Equation 9) has a global optimal solution $\omega_g^* = (\omega_{g1}^*, \omega_{g2}^*, \ldots, \omega_{gn}^*)^T$,,

and $$\omega_{gi}^* = \frac{1 + b_i \sum_{i=1}^{n} 1/a_i - \sum_{i=1}^{n} b_i/a_i}{a_i \sum_{i=1}^{n} 1/a_i}, \quad i = 1,2, \ldots, n, \qquad \text{(A-1)}$$

where $a_i = \sum_{k=1}^{N} H^2(\omega_i^k)$, and $b_i = \sum_{k=1}^{N} H(\omega_i^k)\omega_i^k$.

Proof

When the decision maker DM_k evaluates element E_i , for all i , then there exist $\omega_{ik} > 0$, and $H(\omega_{ik}) = 1$, otherwise $\omega_{ik} = 0$, and $H(\omega_{ik}) = 0$. If there exists at least one decision maker evaluates element E_i , for a group of N decision makers, $\omega_i^1, \omega_i^2, \ldots, \omega_i^N$

that $H(\omega_i^1), H(\omega_i^2), \ldots, H(\omega_i^N)$ are not all zero. The objective function (9) can be solved using the Langrange multiplier method. The Langrangian function is built as follows:

$H(\omega_i^1), H(\omega_i^2), \ldots, H(\omega_i^N)$

$$L(\omega_g, \lambda) = Q(\omega_g) - \lambda g(\omega_g), \qquad \text{(A-2)}$$

where $g(\omega_g) = \sum_{i=1}^{n} \omega_{gi} - 1$, for all i .

Let the fist partial derivatives $\frac{\partial L}{\omega_{gi}}$ and $\frac{\partial L}{\lambda}$ be zero for all i , then we have from (A-2) that:

$$\frac{\partial L}{\partial \omega_{gi}} = \sum_{k=1}^{N} (\omega_i^k - H(\omega_i^k)\omega_{gi})(-H(\omega_i^k)) - \lambda = 0, \qquad \text{(A-3)}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \omega_{gi} - 1 = 0. \qquad \text{(A-4)}$$

Suppose that $a_i = \sum_{k=1}^{N} H^2(\omega_i^k)$, and $b_i = \sum_{k=1}^{N} H(\omega_i^k)\omega_i^k$ for all i , then we could have $a_i > 0$, and $b_i > 0$ since both vectors $\omega_i^1, \omega_i^2, \ldots, \omega_i^N$, and the corresponding vectors $H(\omega_i^1), H(\omega_i^2), \ldots, H(\omega_i^N)$ are not all zero.

Hence, ω_{gi^*} and λ_* for all i can be obtained by solving (A-3) and (A-4) together as given as:

$$\omega_{gi}^* = \frac{1 + b_i \sum_{i=1}^{n} 1/a_i - \sum_{i=1}^{n} b_i/a_i}{a_i \sum_{i=1}^{n} 1/a_i}, \qquad \text{(A-5)}$$

$$\lambda_* = \frac{1 - \sum_{i=1}^{n} b_i / a_i}{\sum_{i=1}^{n} 1/a_i} \quad . \qquad \text{(A-6)}$$

Similarly, since $L(\omega_g, \lambda)$ is twice differentiable at the extremum $\omega_g^* = \left(\omega_{g1}^*, \omega_{g2}^*, \ldots, \omega_{gn}^*\right)^T$, then the bordered Hessian determinant as explained by Avriel (1976) can be given by:

$$D_q(\omega_g^*, \lambda^*) = (-1)^M \begin{vmatrix} \frac{\partial^2 L}{\partial \omega_{g1}^* \partial \omega_{g1}^*} & \frac{\partial^2 L}{\partial \omega_{g1}^* \partial \omega_{g2}^*} & \cdots & \frac{\partial^2 L}{\partial \omega_{g1}^* \partial \omega_{gq}^*} & \frac{\partial g}{\partial \omega_{g1}^*} \\ \vdots & & & & \vdots \\ \frac{\partial^2 L}{\partial \omega_{gq}^* \partial \omega_{g1}^*} & \frac{\partial^2 L}{\partial \omega_{gq}^* \partial \omega_{g2}^*} & \cdots & \frac{\partial^2 L}{\partial \omega_{gq}^* \partial \omega_{gq}^*} & \frac{\partial q}{\partial \omega_{gq}^*} \\ \frac{\partial g}{\partial \omega_{q1}^*} & \frac{\partial g}{\partial \omega_{g2}^*} & \cdots & \frac{\partial g}{\partial \omega_{gq}^*} & 0 \end{vmatrix} , \qquad \text{(A-7)}$$

$$= (-1)^1 \times \begin{vmatrix} \sum_{k=1}^{N} H^{\angle}(\omega_1^{\kappa}) & 0 & \cdots & 0 & 1 \\ 0 & \sum_{k=1}^{N} H_2(\omega_2^k) & \cdots & 0 & 1 \\ \vdots & & & & \vdots \\ 0 & 0 & \cdots & \sum_{k=1}^{N} H^{\angle}(\omega_q^k) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{vmatrix} , \qquad \text{(A-8)}$$

$$= (-1)^{1+1} \times \sum_{i=1}^{q} [1 \Big/ \sum_{k=1}^{N} H^{\angle}(\omega_i^{\kappa})] \cdot \prod_{i=1}^{q} \sum_{k=1}^{N} H^{\angle}(\omega_i^{\kappa}) \quad , \qquad \text{(A-9)}$$

where M is defined as the number of constraint equations,

and $D_q(\omega^*_g, \lambda^*)$ is positive, such that $Q(\omega_g)$ has a strict local minimum at:

$$\omega_g^* = \left(\omega_{g1}^*, \omega_{g2}^*, \ldots, \omega_{gn}^*\right)^T ,$$

where

$$\omega_{gi}^* = \frac{1 + \sum_{i=1}^{n} b_i \Big/ 1\, a_i - \sum_{i=1}^{n} b_i / a_i}{a_i \sum_{i=1}^{n} 1 / a_i} \; , \; i = 1, 2, \ldots, n \, .$$

Since $R = \{(\omega_{g1}, \ldots, \omega_{gn}) \mid \sum_{i=1}^{n} \omega_{gi} = 1, \forall i = 1, 2, \ldots, n\}$ is a nonempty convex set, and $Q(\omega_g)$ is a convex function on R, we can follow that $Q(\omega_{g^*})$ is a global minimum if $Q(\omega_g)$ has a local minimum at ω_{g^*} .