African Journal of Mathematics and Mathematical Sciences ISSN: 3821-435x Vol. 2 (1), pp. 023-037, January, 2014. Available online at www.internationalscholarsjournals.org © International Scholars Journals

Full Length Research Paper

Christ-Obimba tangent, permutation, sequence and series rules in Calculus

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Accepted 02 January, 2014

The aim of this study is to develop a convenient Tangent rule that could be used for determining the area and lengths of sides of a triangle, and also to develop differentiation and integral calculus rules/formulae based on permutation and geometric progression. Christ-Obimba Calculus Rule 1 could be used in determining further/higher derivatives of a function, directly from the parent function, without recourse to the consecutive, previous derivative. For any triangle ($\triangle$) ABC,

$$TanA = \frac{4(Area\ of\ \Delta ABC)}{(\ b^2 + c^2 - a^2\ )}\ ,\ TanB = \frac{4(Area\ of\ \Delta ABC)}{(\ a^2 + c^2 - b^2\ )}\ ,\ \text{and}\ TanC = \frac{4(Area\ of\ \triangle ABC)}{(\ a^2 + b^2 - c^2\ )}.$$

a, b, and c are lengths of the sides of the triangle($\triangle$) ABC (Christ-Obimba Tangent Rules).

If y = ax^n, the formulae, $T_{m+1} = \frac{d^m y}{(dx)^m} = {}^nP_m\, ax^{n-m}$, **and** $T_{m+1} = T_m(n - m + 1)\frac{1}{x}$,
known as Christ-Obimba Calculus Rules 1 and 2, respectively, could be used in calculus of differentiation, for determining derivatives, and further derivatives of mathematical functions. Parent /root mathematical functions, their consecutive derivatives, and further/higher derivatives, form the Christ-Obimba geometric progression sequence in calculus, given by :

ax^n , ${}^nP_1\, ax^{n-1}$, ${}^nP_2\, ax^{n-2}$, ${}^nP_3\, ax^{n-3}$, ------ , ${}^nP_m\, ax^{n-m}$, ${}^nP_n\, ax^{n-n}$, ${}^nP_{n+1}\, ax^{n-(n+1)}$, ----- .

whose sum known as Christ-Obimba geometric progression series in calculus is given by :

$$S_{m+1} = T_1 \frac{\left(1 - \left[(n-m+1)\frac{1}{x}\right]^{m+1}\right)}{\left(1 - \left[(n-m+1)\frac{1}{x}\right]\right)},$$

The Christ-Obimba geometric progression series in calculus could be used to obtain all the derivatives, of the parent function of a finite series, without calculating individual derivatives, singly.

The formula, $T_i = \frac{T_d}{(n-m+1)\times\frac{1}{x}} + C$, **known as Christ-Obimba Calculus Rule 3,**
could be used in calculus of integration,
for determining integrals of functions.
m = the specific derivative (1st, or 2nd or 3rd…….or mth derivative).
n = index power to which the variable x is raised. e.g : x^4 (n = 4).
a = constant = coefficient of the variable term in x^n.
C = constant.
${}^nP_m$ = **number of permutations of m from n dissimilar numbers** $= \frac{n!}{(n-m)!}$, $\quad m < n$.
T_1 =the first term of the geometric progression series of calculus = ax^n.
T_i = the dependent variable y; T_d = the term in the independent variable x.

Key Words: Tangent, calculus, permutation, geometric progression series, derivatives.

INTRODUCTION

Calculus is the mathematical study of rate of change expressed in the unifying themes of differential calculus and integral calculus. The derivative and integral are thus mathematical analysis based on the study of functions and limits, broadly called Calculus (Katz, 1995).

The derivative, $\frac{d[f(x)]}{dx}$, is a measure of how the function, f(x), changes, as its determinant independent variable x, changes. Differentiation is the process of finding a derivative (Talbert et al., 2000).

The tangent to a plane curve given by f(x), at a given point, is the straight line that "just touches" the curve, at that point (Hazewinkel, 2001), and is the $\frac{d[f(x)]}{dx}$, at that point .

Integration is the inverse/(mathematical reverse) of differentiation, and is the measure (area, volume) of a region of an x-y plane (in the case of an area measure), or (x-y-z, 3-dimensional figure) (in the case of volume measure) (Stoer and Bulirsch, 2002).

A sequence is an ordered list of objects (or events), and can be defined as a function whose domain is a countable totally ordered set, such as the natural numbers. Sequences can be finite or infinite (Gaughan, 2009).

The aims of this study includes to develop a convenient Tangent rule/formulae that could be used for determining the area and lengths of sides of a triangle, and also to develop differentiation and integral calculus rules/formulae based on permutation and geometric progression series. I have developed formulae convenient for use in determining derivatives of functions. One of the calculus rules/ formulae could be used in determining further/higher derivatives of a function, directly from the parent function, without recourse to the consecutive, previous derivative.

For example, one could calculate $\frac{d^5y}{(dx)^5}$ from the parent function, y,

without going the whole hog of calculating $\frac{d^2y}{(dx)^2}$, $\frac{d^3y}{(dx)^3}$, and $\frac{d^4y}{(dx)^4}$.

In other words, one does not need to determine $\frac{d^2y}{(dx)^2}$ from $\frac{dy}{dx}$ nor does one need to determine $\frac{d^4y}{(dx)^4}$ from $\frac{d^3y}{(dx)^3}$. The formula enables one to determine the specific derivative directly from the parent or original function.

MATERIALS AND METHODS

1. Christ-Obimba Tangent Rule

$c^2 = a^2 + b^2 - 2ab \text{ Cos } C$: Cosine rule (Bogomolny, 2013).

Figure 1 is triangle ABC with sides a, b, and c.

$\frac{1}{2}abSinC =$ Area of Triangle ABC

2. Christ-Obimba Permutation, Sequence and Series Rules in Calculus.

${}^{n}P_{m} = \frac{n!}{(n-m)!}$. , n is positive , ${}^{n}P_{n} = n!$ (Backhouse and Houldsworth, 1980a).

${}^{n}P_{m} = \frac{-n!}{(n-m)!}$. , n is negative.

${}^{n}P_{m}$ = number of permutations of m from n dissimilar numbers.

The sum of n terms of a geometric progression series is given by

$S_n = b(\frac{1-r^n}{1-r})$(Backhouse and Houldsworth, 1980b).

b = the first term of the geometric progression series.

r = geometric progression common ratio factor.

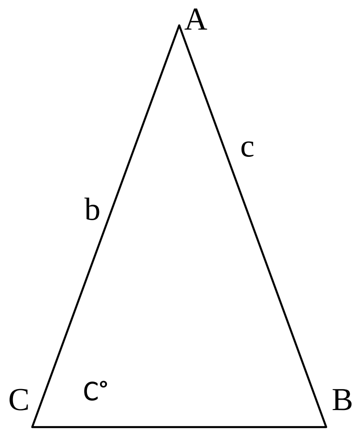

Figure 1. Triangle ABC (△ABC).

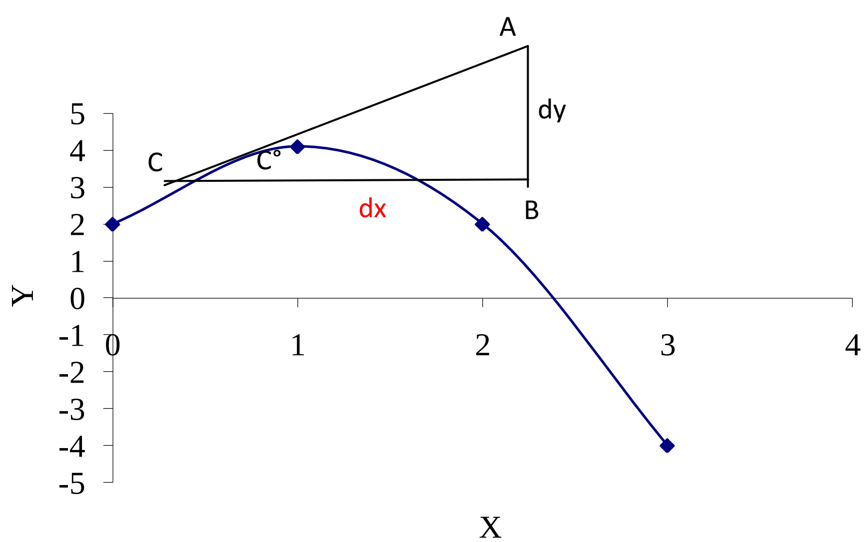

Figure 2 : Y=f (X)

Proof of Christ-Obimba Tangent Rule

Figure 2 is the diagram of the curve, Y = f(x), showing the mini triangle **(△'ABC),** formed by joining the lines δy and δx, and the tangent AC, to the curve.

$C^o = C'$ eqn 1

$\text{Tan } C^o = \text{Tan } C' = \dfrac{dy}{dx}$ eqn 2

To find a value for Tan A, Tan B, and Tan C !

$c^2 = a^2 + b^2 - 2ab \text{ Cos } C$: Cosine rule (Bogomolny, 2013). eqn 3

Dividing eqn 3 by Sin C,

$$\frac{c^2}{SinC} = \frac{a^2 + b^2}{SinC} - \frac{2abCosC}{SinC} \qquad \text{eqn 4.}$$

But $\frac{CosC}{SinC} = \frac{1}{TanC}$ eqn 5.

Substituting the value of $\frac{CosC}{SinC}$ in eqn 4, for its value of eqn 5,

$$\frac{c^2}{SinC} = \frac{a^2 + b^2}{SinC} - \frac{2ab}{TanC}$$

$$\frac{1(c^2)}{SinC} = \frac{1(a^2 + b^2)}{SinC} - \frac{2ab}{TanC}$$

$$\frac{1(c^2)}{SinC} - \frac{1(a^2 + b^2)}{SinC} = -\frac{2ab}{TanC}$$

$$\frac{1(c^2 - a^2 - b^2)}{SinC} = -\frac{2ab}{TanC}$$

Cross multiplying,

$$(c^2 - a^2 - b^2)TanC = -2abSinC \qquad \text{eqn 6.}$$

Dividing eqn 6 by $(c^2 - a^2 - b^2)$,

$$TanC = \frac{-2abSinC}{c^2 - a^2 - b^2} \qquad \text{eqn 7}$$

But $\frac{1}{2}abSinC =$ Area of Triangle ABC eqn 8

-4 ($\frac{1}{2}abSinC) = -2abSinC$ eqn 9

Substituting the value of $\frac{1}{2}abSinC$ in eqn 9, for it's value of eqn 8,

-4 (Area of Triangle ABC) = $-2abSinC$ eqn 10.

Substituting the value of $-2abSinC$ in eqn 7, for it's value of eqn 10,

$$TanC = \frac{-4(Area\ of\ \Delta ABC)}{c^2 - a^2 - b^2}$$

$$TanC = \frac{-4(Area\ of\ \Delta ABC)}{-(-c^2 + a^2 + b^2)} \qquad \text{eqn 11}$$

$$TanC = \frac{4(Area\ of\ \Delta ABC)}{(a^2 + b^2 - c^2)} \qquad \text{eqn 12}$$

$$TanA = \frac{4(Area\ of\ \Delta ABC)}{(b^2 + c^2 - a^2)} \qquad \text{eqn 13}$$

$$TanB = \frac{4(Area\ of\ \Delta ABC)}{(a^2 + c^2 - b^2)} \qquad \text{eqn 14}$$

Equations 12, 13, and 14 are the Christ-Obimba Tangent Rules.
Comparing eqns. 2 and 12,

$$TanC = TanC' = \frac{dy}{dx} = \frac{4(Area\ of\ \Delta ABC)}{(a^2 + b^2 - c^2)} \qquad \text{eqn 15}$$

Q.E.D

Proof of Christ-Obimba Calculus Rule 1 (Permutation), Geometric Progression Sequence and Series in Calculus

Let $y = ax^n$

A geometric progression sequence is given by :

$$y, \frac{dy}{dx}, \frac{d^2y}{(dx)^2}, \frac{d^3y}{(dx)^3}, \text{-----} \frac{d^my}{(dx)^m}, \text{------}, \frac{d^ny}{(dx)^n}, \frac{d^{n+1}y}{(dx)^{n+1}},$$

which resolves to :

$$ax^n,\ {}^nP_1\, ax^{n-1}, {}^nP_2\, ax^{n-2}, {}^nP_3\, ax^{n-3}, \text{------},\ {}^nP_m\, ax^{n-m}, ---, {}^nP_n\, ax^{n-n},\ {}^nP_{n+1}\, ax^{n-(n+1)}, \text{-----}.$$

and gives a geometric progression series :

$$ax^n + {}^nP_1\, ax^{n-1} + {}^nP_2\, ax^{n-2} + {}^nP_3\, ax^{n-3} + \text{------} + {}^nP_m\, ax^{n-m} + {}^nP_n\, ax^{n-n} + {}^nP_{n+1}\, ax^{n-(n+1)} + \text{-----}.$$

The first term of the geometric progression series is the parent function, $y = ax^n$.

The second term of the geometric progression series, is the first derivative, $\frac{dy}{dx}$, of the parent function, and is ${}^nP_1\, ax^{n-1}$.

The third term of the geometric progression series, is the second derivative, $\frac{d^2y}{(dx)^2}$, of the parent function, and is ${}^nP_2\, ax^{n-2}$.

The $m + 1$ term of the geometric progression series, is the mth derivative, $\frac{d^my}{(dx)^m}$, of the parent function, and is ${}^nP_m\, ax^{n-m}$.

The $n + 1$ term of the geometric progression series, is the nth derivative, $\frac{d^ny}{(dx)^n}$, of the parent function, and is ${}^nP_n\, ax^{n-n}$.

The $n + 2$ term of the geometric progression series, is the $(n + 1)$th derivative, $\frac{d^{n+1}y}{(dx)^{n+1}}$, of the parent function, and is ${}^nP_{n+1}\, ax^{n-(n+1)} = 0$ (if n is +ve, and is a whole no.).

This is the last term of the geometric progression series (if n is +ve, and is a whole no.), and is so, because the term preceding it, is a constant, and the differential coefficient of any constant (number) is 0 (zero).

The total number of terms in the geometric progression finite series is given by n + 2 (if n is +ve, and is a whole no.).

The m + 1 term of the geometric progression series, gives the value of the mth derivative, which could be any specified derivative of the parent function., and is given by : $\mathbf{T_{m+1} = {}^nP_m\, ax^{n-m}}$. ($n \neq 0$), hereafter known and referred to as Christ-Obimba Calculus Rule 1 (One).

m = the specific derivative (1st, or 2nd or 3rd…….or mth derivative).

n = index power to which the variable x is raised. For example, x^4 (n = 4), x^{-3} (n = -3),

x^4 (n = 4), $x^{\frac{1}{2}}$ (n = $\frac{1}{2}$), $x^{-\frac{1}{3}}$ (n = $-\frac{1}{3}$)

a = constant = coefficient of the variable term in x^n.

Q.E.D

Proof of Calculus rule 2, and further proof of the Christ-Obimba Geometric Progression Sequence and Series in Calculus

Let a function be given by $y = 2x^4$.

$\mathbf{T_1}$ is the parent function, and is given by $T_{m+1} = {}^nP_m\, ax^{n-m}$.

m = 0, n = 4, a = 2

$\mathbf{T_{m+1} = T_{0+1} = T_1} = 2x^4$ = y = the parent function.

$\mathbf{T_2}$ is the second term of the geometric progression sequence, but the first derivative of the parent function.

m = 1, n = 4, a = 2.

$\mathbf{T_{m+1} = T_{1+1} = T_2 =}\ {}^4P_1 \times 2x^{4-1} = 4 \times 2x^{4-1} = 8x^3$.

$\mathbf{T_3}$ is the third term of the geometric progression sequence, but the second derivative of the parent function.

m = 2, n = 4, a = 2.

$\mathbf{T_{m+1}} = \mathbf{T_{2+1}} = \mathbf{T_3} = {}^4P_2 \times 2x^{4-2} = \frac{4!}{(4-2)!} \times 2x^2 = \frac{4!}{2!} \times 2x^2 = \frac{4 \times 3 \times 2 \times 1}{1 \times 2} \times 2x^2$
$= \frac{24}{2} \times 2x^2 = 24x^2$

$\mathbf{T_4}$ is the fourth term of the geometric progression sequence, but is the third derivative of the parent function.
m = 3, n = 4, a = 2.
$\mathbf{T_{m+1}} = \mathbf{T_{3+1}} = \mathbf{T_4} = {}^4P_3 \times 2x^{4-3} = \frac{4!}{(4-3)!} \times 2x^1 = \frac{4!}{1!} \times 2x = \frac{4 \times 3 \times 2 \times 1}{1} \times 2x$
$= 24 \times 2x = 48x$

$\mathbf{T_5}$ is the fifth term of the geometric progression sequence, but is the fourth derivative of the parent function.
m = 4, n = 4, a = 2.
$\mathbf{T_{m+1}} = \mathbf{T_{4+1}} = \mathbf{T_5} = {}^4P_4 \times 2x^{4-4} = \frac{4 \times 3 \times 2 \times 1}{1} \times 2 = 24 \times 2$
$= 48$
(Hints : ${}^nP_n = n!$, ${}^4P_4 = 4!$, and $x^0 = 1$).

$\mathbf{T_6}$ is the sixth term of the geometric progression sequence, but is the fifth, and last derivative of the parent function.
m = 5, n = 4, a = 2.
$\mathbf{T_{m+1}} = \mathbf{T_{5+1}} = \mathbf{T_6} = {}^4P_5 \times 2x^{4-5} = 0 \times 2x^{-1} = 0$
(Hints: if n is +ve, and a whole no., and m > n, ${}^nP_m = 0$).

Therefore, the sequence : T_1, T_2, T_3, T_4, T_5, and T_6, is $2x^4$, $8x^3$, $24x^2$, 48x, 48, 0, and is a geometric progression sequence, given by :

$\mathbf{ax^n, \ {}^nP_1\, ax^{n-1}, {}^nP_2\, ax^{n-2}, {}^nP_3\, ax^{n-3}, ------, \ {}^nP_m\, ax^{n-m}, ---, {}^nP_n\, ax^{n-n}, \ {}^nP_{n+1}\, ax^{n-(n+1)}}$, ----- .
hereafter known and referred to as the **Christ-Obimba geometric progression sequence in calculus** of which the common ratio factor is :

$$\mathbf{(n-m+1)\times\frac{1}{x}}.$$

In otherwords, in order to obtain T_{m+1} from T_m (ie to find a new term in the geometric progression sequence from a previous consecutive term), the latter term is multiplied by the common ratio factor, $\mathbf{(n-m+1)\times\frac{1}{x}}$, to give **:**

$\mathbf{T_{m+1} = T_m\,(n-m+1)\times\frac{1}{x}}$ (x is the dependent variable) hereafter known and referred to as

Christ-Obimba calculus rule 2 eqn 1

To obtain the first derivative, $\frac{dy}{dx}$, from the parent function, $y = 2x^4$, the formula,

Christ-Obimba calculus rule 2 , $\mathbf{T_{m+1} = T_m\,(n-m+1)\times\frac{1}{x}}$, is applied as follows :
m = 1 (an indication that it is the first derivative, though, the second term in the sequence) eqn 2
n = 4 eqn 3
Substituting the values of m and n in equation 1, for their values of equations 2 and 3,
$T_{m+1} = T_{1+1} = T_2 = T_1\,(4-1+1)\times\frac{1}{x} = T_1\,(4)\times\frac{1}{x}$ eqn 4
T_1 is the parent function, $2x^4$, and the first term of the geometric progression sequence.
Substituting the value of T_1 in equation 4, for its value of $2x^4$,
$T_{m+1} = T_2 = 2x^4 \times(4)\times\frac{1}{x} = 8x^{4-1} = 8x^3$ eqn 5
$T_1 = 8x^3$, is the value of the first derivative obtained from the parent function, $y = 2x^4$, and is also the second term of the geometric progression sequence.

To obtain the third derivative, $\frac{d^3y}{(dx)^3}$, from the previous consecutive function, $\frac{d^2y}{(dx)^2}$,

which is the second derivative, the formulae, $\mathbf{T_{m+1} = T_m\ (n\text{-}m+1)\times \frac{1}{x}}$ (eqn 1), is applied.
In otherwords, for the third derivative, T_3 (though, the fourth term in the series),
m = 3 (an indication that it is the third derivative, though, the fourth term in the sequence) eqn 6
n = 4 eqn 7
Substituting the values of m and n in equation 1, for their values of equations 6 and 7,

$$T_{m+1} = T_{3+1} = T_4 = T_3\ (4\text{-}3+1)\times \frac{1}{x} = T_3\ (2)\times \frac{1}{x} \qquad \text{eqn 8}$$

T_3 is $24x^2$, and is the second derivative ($\frac{d^2y}{(dx)^2}$), but the third term in the geometric progression sequence.

Substituting the value of T_3 in equation 8, for its value of $24x^2$,

$$T_{m+1} = T_4 = 24x^2 \times (2)\times \frac{1}{x} = 48x^{2-1} = 48x \qquad \text{eqn 9}$$

$T_4 = 48x$, is the value of the third derivative of the parent function, obtained from the second derivative of the parent function, and is also the fourth term of the geometric progression sequence.

Proof of Christ-Obimba Calculus Rule 3

$$\text{Let } y = \int ax^n\, dx, \qquad \text{eqn 1}$$

The formula, $T_{m+1} = T_m\ (n\text{-}m+1)\times \frac{1}{x}$, which I developed, equates to the formula, $T_d = T_i\ (n\text{-}m+1)\times \frac{1}{x}$, which I have also developed, for the purpose of integration,

and is applied, as follows : $T_i = \frac{T_d}{(n\text{-}m+1)\times \frac{1}{x}} + C$ eqn 2

T_i = the dependent variable y eqn 3
T_d = the term in the independent variable x = ax^n eqn 4
m = 0 eqn 5
For the purpose of integration, for obtaining the integral of a given function, m is always equal to 0.
n = power index of the dependent variable x eqn 6

If $\mathbf{y = \int ax^n\, dx,\ T_i = \frac{T_d}{(n\text{-}m+1)\times \frac{1}{x}} + C,}$ hereafter known and referred to as

Christ-Obimba Calculus Rule 3

$T_i = y = \int ax^n$
$T_d = ax^n$
m = 0
a = constant
C = constant
Q.E.D.
The sum of n terms of a geometric progression series is given by
$S_n = b(\frac{1-r^n}{1-r})$(Backhouse and Houldsworth, 1980b).
b = the first term of the geometric progression series.
r = geometric progression common ratio factor.
The sum of the geometric progression series obtained from my Christ-Obimba geometric progression sequence in calculus is given by :

$$ax^n + {}^nP_1\, ax^{n-1} + {}^nP_2\, ax^{n-2} + {}^nP_3\, ax^{n-3} + \text{------} + {}^nP_m\, ax^{n-m} + {}^nP_n\, ax^{n-n} + {}^nP_{n+1}\, ax^{n-(n+1)} + \text{-----}\,.$$

and is $S_{m+1} = T_1 \dfrac{\left(1-\left[(n-m+1)\frac{1}{x}\right]^{m+1}\right)}{\left(1-\left[(n-m+1)\frac{1}{x}\right]\right)}$, hereafter known and referred to as the sum of terms of the

Christ-Obimba geometric progression series in calculus.

m = the specific derivative (1st, or 2nd or 3rd…….or mth derivative).
n = index power to which the variable x is raised. For example, x^4 (n = 4), x^{-3} (n = -3), x^4 (n = 4)
T_1 =the first term of the geometric progression series of calculus = ax^n.

The geometric progression common ratio factor = $(n-m+1)\times\frac{1}{x}$.

Q.E.D.

RESULTS

Application of Christ-Obimba Tangent Rule

Figure 3 shows a curve Y= f(X), whose tangent AC forms angle C with the horizontal of length m units. The tangent is joined to the horizontal side by a line AB, to form a right angled triangle. BD is a line drawn from angle B to meet AC at D. Line CD = m.

Prove that the area of triangle ABC = $\frac{1}{2}m^2 TanC^\circ$ unit^2

Hence show that the area of triangle ABC = $\frac{1}{2}m^2\frac{dy}{dx}$ unit^2

If Y = 2 + 4x – 2x^2 , find the area of triangle ABC in terms of m, given that the tangent $\overline{AC}$ meets the curve, Y = 2 + 4x – 2x^2 at the point x = 0.5.

Solution

$\frac{\overline{AB}}{m}$ is a function of $\frac{dy}{dx}$ = Tan C° eqn 1

$$TanC' = \frac{4(Area\ of\ \Delta ABC)}{m^2 + (\overline{AC})^2 - (\overline{AB})^2} \qquad \text{eqn 2.}$$

$$(\overline{AC})^2 = [m + (\overline{AD})]^2 \qquad \text{eqn 3}$$

By Pythagora's theorem,

$$(\overline{AB})^2 = (\overline{AC})^2 - m^2 \qquad \text{eqn 4.}$$

Substituting the value of $(\overline{AB})^2$ in eqn 2, for it's value of eqn 4,

$$TanC^\circ = \frac{4(Area\ of\ \Delta ABC)}{m^2 + (\overline{AC})^2 - [(\overline{AC})^2 - m^2]}$$

$$TanC^\circ = \frac{4(Area\ of\ \Delta ABC)}{m^2 + m^2}$$

$$TanC^\circ = \frac{4(Area\ of\ \Delta ABC)}{2m^2}$$

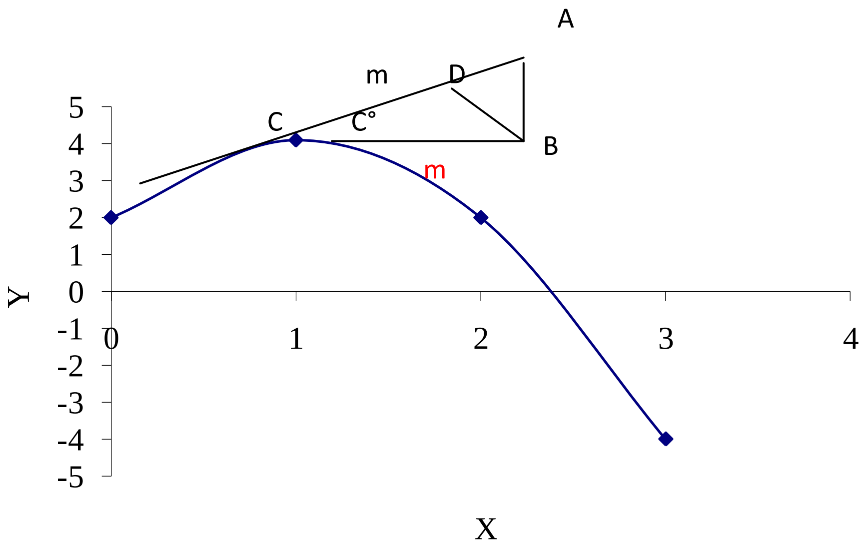

Figure 3. Y=f (X)

$$TanC^{\circ} = \frac{2(Area\ of\ \Delta ABC)}{m^2}$$ eqn 5

Cross multiplying,

$$m^2 TanC^{\circ} = 2\,Area\,of\ \Delta ABC$$

$$Area\,of\ \Delta ABC = \frac{1}{2} m^2 TanC^{\circ}$$ eqn 6

Q.E.D.

Substituting the value of $TanC^{\circ}$ in eqn 6, for its value of eqn 1,

$$Area\,of\ \Delta ABC = \frac{1}{2} m^2 \frac{dy}{dx}$$ eqn 7

Q.E.D

If Y = 2 + 4x – 2x^2 eqn 8,

$$\frac{dy}{dx} = 4 - 4x$$ eqn 9

Substituting the value of x in eqn 9, for its value of 0.5,

$$\frac{dy}{dx} = 4 - 4 \times 0.5 = 4 - 2 = 2$$ eqn 10

Substituting the value of $\frac{dy}{dx}$ in eqn 7, for its value of eqn 10,

$$Area\,of\ \Delta ABC = \frac{1}{2} m^2 \times 2 = m^2\ unit^2$$

$$Area\,of\ \Delta ABC = m^2\ unit^2$$ **Answer** eqn 11

Application of Christ-Obimba Calculus Rule 1 (Permutation Rule) in Calculus of Differentiation

1) <u>n is a whole positive number, equal to or greater than 1 (n ≥ 1).</u>
Find the second and fourth derivatives of the function, $y = 3x^7 + 2x^6$.
Solution using Christ-Obimba Calculus Rule One (1)
$y = 3x^7 + 2x^6$ eqn 1
For the purpose of determining the second derivative,
m = 2 eqn 2
mth = 2nd (second) derivative but third (m + 1) term.
for the term $3x^7$,
$n_1 = 7$, eqn 3
$a_1 = 3$, eqn 4
for the term $2x^6$,
$n_2 = 6$, eqn 5
$a_2 = 2$, eqn 6

$$n_1P_m = {}^7P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7\times6\times5\times4\times3\times2\times1}{1\times2\times3\times4\times5} = 7\times6 = 42 \qquad \text{eqn 7}$$

$${}^{n2}P_m = {}^6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6\times5\times4\times3\times2\times1}{1\times2\times3\times4} = 6\times5 = 30 \qquad \text{eqn 8}$$

$$\text{But, } T_{m+1} = T_{2+1} = T_3 = \frac{d^2y}{(dx)^2} = f''(x) = {}^{n1}P_m a_1 x^{n_1\text{-m}} + {}^{n2}P_m a_2 x^{n_2\text{-m}}. \qquad \text{eqn 9}$$

Substituting the values of m, n_1 , a_1, n_2 , a_2 , ${}^{n1}P_m$, and ${}^{n2}P_m$ in equation 9, for their values of equations 2, 3, 4, 5, 6, 7, and 8, respectively,

$$T_{m+1} = T_3(\text{third term}) = \frac{d^2y}{(dx)^2} = f''(x) = 42\times3x^{7-2} + 30\times2x^{6-2}$$

$= \mathbf{126x^5 + 60x^4}$. **(Answer)** eqn 10
For the purpose of determining the fourth derivative,
m = 4 eqn 11
mth = 4th (fourth) derivative but fifth (m + 1) term.
for the term $3x^7$,
$n_1 = 7$, eqn 12
$a_1 = 3$, eqn 13
for the term $2x^6$,
$n_2 = 6$, eqn 14
$a_2 = 2$, eqn 15

$$n_1P_m = {}^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7\times6\times5\times4\times3\times2\times1}{1\times2\times3} = 7\times6\times5\times4 = 840. \qquad \text{eqn 16}$$

$$n_2P_m = {}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times2\times1}{1\times2} = 6\times5\times4\times3 = 360 \qquad \text{eqn 17}$$

$$\text{But, } T_{m+1} = T_{4+1} = T_5 = \frac{d^4y}{(dx)^4} = f''(x) = {}^{n1}P_m a_1 x^{n_1\text{-m}} + {}^{n2}P_m a_2 x^{n_2\text{-m}}. \qquad \text{eqn 18}$$

Substituting the values of m, n_1 , a_1, n_2 , a_2 , ${}^{n1}P_m$, and ${}^{n1}P_m$ in equation 18, for their values of equations 11, 12, 13, 14, 15, 16, and 17, respectively,

$$T_{m+1} = T_5(\text{fifth term}) = \frac{d^4y}{(dx)^4} = f''''(x) = 840\times3x^{7-4} + 360\times2x^{6-4}$$

$= \mathbf{2520x^3 + 720x^2}$. **(Answer)** eqn 19
Equations 18 and 19 are the two steps that suffice as the solution. The other solution steps are shown for the purpose of being elaborate.

2) <u>n is positive, and is less than 1 (0 < n < 1).</u>

Find the second and third derivatives of the function, $y = x^{\frac{1}{2}}$

Solution using Christ-Obimba Calculus Rule One (1)
$y = x^{\frac{1}{2}}$ eqn 1
For the purpose of determining the second derivative,

m = 2 eqn 2
mth = 2nd (second) derivative, but third (m + 1) term
n = $\frac{1}{2}$ eqn 3
a = 1 eqn4
$^{n}P_{m} = {}^{1/2}P_{2} = \frac{\frac{1}{2}(\frac{1}{2} - 1)}{1} = \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}$ eqn 5
But, $T_{m+1} = T_{2+1} = T_3 = \frac{d^2y}{(dx)^2} = f''(x) = {}^{n}P_{m}\, ax^{n-m}$ eqn 6
Substituting the values of m, n, a, and $^{n}P_{m}$ in equation 6,
for their values of equations 2, 3, 4, and 5, respectively,
$T_{m+1} = T_3(\text{third term}) = \frac{d^2y}{(dx)^2} = f''(x) = -\frac{1}{4} \times 1 \times x^{\frac{1}{2}-2} = \mathbf{-\frac{1}{4}\, x^{-\frac{3}{2}}}$ $(\boldsymbol{Answer})$. eqn 7
For the purpose of determining the third derivative of the function, y = $x^{\frac{1}{2}}$,

m = 3 eqn 8
mth = 3rd (third) derivative but fourth (m + 1) term
n = $\frac{1}{2}$ eqn 9
a = 1 eqn10

$^{n}P_{m} = {}^{1/2}P_{3} = \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{1} = \frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} = +\frac{3}{8}$ eqn 11
But, $T_{m+1} = T_{3+1} = T_4 = \frac{d^3y}{(dx)^3} = f''(x) = {}^{n}P_{m}\, ax^{n-m}$ eqn 12
Substituting the values of m, n, a, and $^{n}P_{m}$ in equation 12,
for their values of equations 8, 9, 10, and 11, respectively,
$T_{m+1} = T_4(\text{fourth term}) = \frac{d^3y}{(dx)^3} = f'''(x) = \frac{3}{8} \times 1 \times x^{\frac{1}{2}-3} = \frac{3}{8}\, x^{-\frac{5}{2}}$
$= \mathbf{\frac{3}{8}\, x^{-2\frac{1}{2}}}$ $(\boldsymbol{Answer})$ eqn 13
Equations 12 and 13 are the two steps that suffice as the solution. The other solution steps are shown for the purpose of being elaborate.

3) <u>n is a negative whole no. (n < 0).</u>

Find the first, second, and third derivatives of the function, y = $2x^{-3}$.

Solution using Christ-Obimba Calculus Rule One (1)

y = $2x^{-3}$. eqn 1
a) For the purpose of determining the first derivative,
m = 1 eqn 2
mth = 1st (first) derivative, but the second (m + 1) term
n = -3 eqn 3
a = 2 eqn 4
$^{n}P_{m} = {}^{-3}P_{1} = -3$ (proof : $^{n}P_{1} = n$) eqn 5
But, $T_{m+1} = T_{1+1} = T_2 = \frac{dy}{dx} = f'(x) = {}^{n}P_{m}\, ax^{n-m}$. eqn 6
Substituting the values of m, n, a, and $^{n}P_{m}$ in equation 6,
for their values of equations 2, 3, 4, and 5, respectively,
$T_{m+1} = T_2(\text{second term}) = \frac{d^2y}{(dx)^2} = f'(x) = -3 \times 2\, x^{-3-1} = \mathbf{-6\, x^{-4}}$ $(\boldsymbol{Answer})$. eqn 7
For the purpose of determining the second derivative, y = $2x^{-3}$
m = 2 eqn 8
mth = 2nd (second) derivative
n = -3 eqn 9

a = 2 eqn 10

$^{n}P_{m} = {}^{-3}P_{2} = \frac{-3 \times -4}{(3-2)!} = \frac{12}{1!} = 12$ eqn 11

But, $T_{m+1} = T_{2+1} = T_3 = \frac{d^2y}{(dx)^2} = f''(x) = {}^{n}P_{m}\,ax^{n-m}$. eqn 12

Substituting the values of m, n, a, and $^{n}P_{m}$ in equation 12,
for their values of equations 8, 9, 10, and 11, respectively,

$T_{m+1} = T_3 = \frac{d^2y}{(dx)^2} = f''(x) = 12 \times 2\,x^{-3\,-2} = \mathbf{24\,x^{-5}}$ **(Answer).** eqn 13

For the purpose of determining the third derivative, $y = 2x^{-3}$

m = 3 eqn 14

mth = 3rd (third) derivative, but the fourth (m +1) term.

n = -3 eqn 15

a = 2 eqn 16

$^{n}P_{m} = {}^{-3}P_{3} = -3! = -3 \times -4 \times -5 = -60$ eqn 17

But, $T_{m+1} = T_{3+1} = T_4 = \frac{d^3y}{(dx)^3} = f'''(x) = {}^{n}P_{m}\,ax^{n-m} = {}^{-3}P_{3} \times 2x^{-3-3}$. eqn 18

Substituting the values of m, n, a, and $^{n}P_{m}$ in equation 18,
for their values of equations 14, 15, 16, and 17, respectively,

$T_{m+1} = T_4$ (fourth term)= $\frac{d^3y}{(dx)^3} = f'''(x) = -60 \times 2\,x^{-3\,-3} = \mathbf{-120\,x^{-6}}$ **(Answer).** eqn 19

Equations 18 and 19 are the two steps that suffice as the solution. The other solution steps are shown for the purpose of being elaborate.

4) <u>n is negative, and a fraction.</u>

Find the second derivative of the function, $y = 3x^{-\frac{1}{3}}$.

Solution using Christ-Obimba Calculus Rule One (1)

m = 2 eqn 1

mth = 2nd (second) derivative, but third (m + 1) term.

$n = -\frac{1}{3}$ eqn 2

$^{n}P_{m} = {}^{-1/3}P_{2} = -\frac{1}{3}(-\frac{1}{3} - 1) = -\frac{1}{3} \times -\frac{4}{3} = +\frac{4}{9}$ eqn 3

a = 3 eqn 4

$T_{m+1} = T_{2+1} = T_3$(third term)= $\frac{d^my}{(dx)^m} = \frac{d^2y}{(dx)^2} = f''(x) = {}^{n}P_{m}\;ax^{n-m}$ eqn 5

Substituting the values of m, n, $^{n}P_{m}$, and a in equation 5, for their values of equations 1, 2, 3 and 4 respectively,

$T_{m+1} = T_3$(third term)= $\frac{d^my}{(dx)^m} = \frac{d^2y}{(dx)^2} = f''(x) = {}^{-1/3}P_{2} \times 3x^{-\frac{1}{3}-2}$

$= \frac{4}{3}x^{-2\frac{1}{3}} = \mathbf{1\frac{1}{3}x^{-2\frac{1}{3}}}$ **(Answer)** eqn 6

Application of the Christ-Obimba Geometric Progression Series in Calculus.
To determine the sum of the first two terms of the finite geometric progression series of calculus, derived from the function, $y = 2x^4$.

m = 1st derivative, but sum of (m+1) term.

$S_{m+1} = S_{1+1} = S_2$.

T_1 =the first term of the geometric progression series of calculus = $2x^4$ = the value of the parent function, (y).

n = 4.

$$S_{m+1} = T_1 \frac{\left(1-\left[(n-m+1)\frac{1}{x}\right]^{m+1}\right)}{\left(1-\left[(n-m+1)\frac{1}{x}\right]\right)} = S_2 = 2x^4 \times \frac{\left(1-\left[(4-1+1)\frac{1}{x}\right]^{1+1}\right)}{\left(1-\left[(4-1+1)\frac{1}{x}\right]\right)}$$

$$= 2x^4 \times \frac{(1-\left[(4)\frac{1}{x}\right]^2)}{(1-\left[(4)\frac{1}{x}\right])} = 2x^4 \times \frac{(1-\left[\frac{4^2}{x^2}\right])}{(1-\left[\frac{4}{x}\right])} = 2x^4 \times \frac{(1-\frac{4}{x})(1+\frac{4}{x})}{(1-\frac{4}{x})}$$

$$= 2x^4 \times \left(1+\frac{4}{x}\right) = 2x^4 + 2x^4 \times \frac{4}{x} = 2x^4 + \frac{8x^4}{x} = 2x^4 + 8x^{4-1} = 2x^4 + 8x^3$$

.

$S_2 = 2x^4 + 8x^3$.

The sum of the first two terms (S_2) of the finite geometric progression series of calculus, derived from the function, y = $2x^4$, is equal to $2x^4 + 8x^3$.

Application of Christ-Obimba Calculus Rule 3 in Integration

$y = \int 6x^2\, dx$ eqn 1

To obtain the value of $\int 6x^2\, dx$,

The formula, $T_i = \dfrac{T_i}{(n-m+1)\times\frac{1}{x}} + C$, is applied, as follows eqn 2

T_i = the dependent variable y eqn 3

T_d = the term in the independent variable x = $6x^2$ eqn 4

m = 0 eqn 5

For the purpose of integration, for obtaining the integral of a given function, m is always equal to 0.

n = power index of the dependent variable x eqn 6

Substituting the values of T_i, T_d, m, and n, in equation 2, for their values of equations 3,4, 5, and 6,

$$T_i = y = \int 6x^2\, dx = \frac{T_d}{(n-m+1)\times\frac{1}{x}} + C = \frac{6x^2}{(2-0+1)\times\frac{1}{x}} + C$$

$$y = \int 6x^2\, dx = \frac{6x^2}{3\times\frac{1}{x}} + C = \frac{6x^2}{\frac{3}{x}} + C = \frac{6x^2}{3} \times x + C = 2x^2 \times x + C = 2x^3 + C$$ eqn 7

$y = \int 6x^2\, dx = \mathbf{2x^3+C}$ **(Answer)** eqn 8

DISCUSSION

Calculus can be used in solving problems associated with trigonometric functions of triangles, areas and sides of triangles considered in relation to curves.

The area of the triangle ABC, shown in figure 3, was calculated, using only, the values of m unit (the length of the adjacent side of the triangle ABC), and the $\frac{dy}{dx}$ of the curve.

Results of the solution to the problem, $\dfrac{d^4(3x^7 + 2x^6)}{(dx)^4}$,

obtained using the Christ-Obimba Calculus Rule 1, is compared with the solution to the same problem, obtained, using the conventional formula, $\frac{dy}{dx} = nax^{n-1}$ (Cirillo, 2007), if y = ax^n, as follows :

y = $3x^7 + 2x^6$. eqn1

$\frac{dy}{dx} = 7 \times 3x^{7-1} + 6 \times 2x^{6-1} = 21x^6 + 12x^5$ *eqn 2*

$\frac{d(\frac{dy}{dx})}{dx} = \frac{(d^2y)}{(dx)^2} = 6 \times 21x^{6-1} + 5 \times 12x^{5-1} = 126x^5 + 60x^4$ *eqn 3*

$\frac{d(\frac{d^2y}{(dx)^2})}{dx} = \frac{(d^3y)}{(dx)^3} = 126 \times 5x^{5-1} + 60 \times 4x^{4-1} = 630x^4 + 240x^3$ *eqn 4*

$\frac{d(\frac{d^3y}{(dx)^3})}{dx} = \frac{(d^4y)}{(dx)^4} = 630 \times 4x^{4-1} + 240 \times 3x^{3-1} = 2520x^3 + 720x^2$ *eqn 5*

The latter solution (four lines of solution) obtained, using the conventional formula, confirms the former solution (two effective lines of solution) obtained using my Christ-Obimba Calculus Rule 1. Refer to: Application of Christ-Obimba Calculus Rule 1 (Permutation Rule) in Calculus of Differentiation (n is a whole positive number, equal to or greater than 1 (n ≥ 1)).

Results of the solution to the problem, $\frac{d^3(x^{\frac{1}{2}})}{(dx)^3}$,

obtained using the Christ-Obimba Calculus Rule 1, is compared with the solution to the same problem,

obtained, using the conventional formula, $\frac{dy}{dx} = nax^{n-1}$, if y = ax^n, as follows :

$y = x^{\frac{1}{2}}$. eqn 1

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$ *eqn 2*

$\frac{d(\frac{dy}{dx})}{dx} = \frac{(d^2y)}{(dx)^2} = -\frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{4}x^{-1\frac{1}{2}} = -\frac{1}{4}x^{-\frac{3}{2}}$ *eqn 3*

$\frac{d(\frac{d^2y}{(dx)^2})}{dx} = \frac{(d^3y)}{(dx)^3} = -\frac{3}{2} \times -\frac{1}{4}x^{-\frac{3}{2}-1} = \frac{3}{8}x^{-2\frac{1}{2}}$ *eqn 4*

The latter solution (three lines of solution) obtained, using the conventional formula, confirms the former solution (two effective lines of solution) obtained using my Christ-Obimba calculus rule 1. Refer to : Application of Christ-Obimba Calculus Rule 1 (Permutation Rule) in Calculus of Differentiation (n is positive, and is less than 1 (0 < n < 1)).

Results of the solution to the problem, $\frac{d^3(2x^{-3})}{(dx)^3}$,

obtained using the Christ-Obimba Calculus Rule 1, is compared with the solution to the same problem,

obtained, using the conventional formula, $\frac{dy}{dx} = nax^{n-1}$, if y = ax^n, as follows :

$y = 2x^{-3}$ eqn 1

$\frac{dy}{dx} = -3 \times 2x^{-3-1} = -6x^{-4}$ eqn 2

$\frac{d(\frac{dy}{dx})}{dx} = \frac{(d^2y)}{(dx)^2} = -4 \times -6x^{-4-1} = 24x^{-5}$ eqn 3

$\frac{d(\frac{d^2y}{(dx)^2})}{dx} = \frac{(d^3y)}{(dx)^3} = -5 \times 24x^{-5-1} = -120x^{-6}$ eqn 4

The latter solution (three lines of solution) obtained, using the conventional formula, confirms the former solution (two effective lines of solution) obtained using my Christ-Obimba Calculus Rule 1. Refer to: Application of Christ-Obimba Calculus Rule 1 (Permutation Rule) in Calculus of Differentiation (n is a negative whole no. (n < 0)).

If n is +ve, and a whole number, the Christ-Obimba Geometric Progression Series of calculus, is a finite one, and the maximum value of the sum (S_{m+1}), of all the terms of the series = S_{n+2}.

For example, if $T_1 = y = 2x^4$, the maximum value of the sum (S_{m+1}), of all the terms of the finite geometric progression series of calculus, derived from the function (y) is S_{n+2}.

But n = 4.
Therefore, $S_{n+2} = S_{4+2} = S_6$
In other words, the maximum value of the sum (S_{m+1}), of all the terms of the finite geometric progression series of calculus, derived from the function ($y = 2x^4$), is the sum of the first six terms of the series.
Invariably, $S_{m+1} = S_6$, m = 5: implying that the sum of the first five consecutive derivatives of the function, (y), and the function, (y), gives the maximum value of the sum (S_{m+1}), of all the terms of the finite geometric progression series of calculus, derived from the function ($y = 2x^4$).
The Christ-Obimba geometric progression series in calculus, like the Taylor Series, and Maclaurin Series used in calculus, is of peculiar importance. The Christ-Obimba geometric progression series in calculus could be used to obtain all the derivatives, of the parent function, of a finite series, or as many of the derivatives as one would want to obtain, of the parent function, of a finite or infinite series, without calculating individual derivatives, singly. Taylor's Series is a representation of a function as an infinite sum of terms which are calculated from the values of the function's derivatives at a single point. Maclaurin Series is Taylor's Series, centered at zero (Odibat and Shawagfeh, 2007).

CONCLUSION

For any triangle ($\triangle$) ABC,
$TanA = \frac{4(Area\ of\ \Delta ABC)}{(\ b^2 + c^2 - a^2\ )}$, $TanB = \frac{4(Area\ of\ \Delta ABC)}{(\ a^2 + c^2 - b^2\ )}$, and $TanC = \frac{4(Area\ of\ \Delta ABC)}{(\ a^2 + b^2 - c^2\ )}$, and are known as Christ-Obimba Tangent Rules. The new and main contributions to knowledge associated with the new tangent rules are that they could be used as complements of sine and cosine rules, in solving problems that border on trigonometric functions of triangles, areas, and sides of triangles considered in relation to curves and calculus.
The formulae, $T_{m+1} = {}^nP_m\, ax^{n-m}$, and $T_{m+1} = T_m(n-m+1)\times\frac{1}{x}$, could be used in calculus of differentiation, for determining derivatives, and further derivatives of mathematical functions, and are known as Christ-Obimba Calculus Rules 1 and 2, respectively. Another major contribution to knowledge is that Christ-Obimba Calculus Rule 1 could be used in determining further/higher derivatives of a function, directly from the parent function, without recourse to the consecutive, previous derivative, and furthermore, permutation, using even fractions and negative numbers, have been effectively employed in determining derivatives and higher derivatives of parent functions. Parent/root mathematical functions, their consecutive derivatives, and further/higher derivatives, form the Christ-Obimba geometric progression sequence in calculus, given by:
$ax^n,\ {}^nP_1\, ax^{n-1}, {}^nP_2\, ax^{n-2}, {}^nP_3\, ax^{n-3}$,------, ${}^nP_m\, ax^{n-m}$, – – –, ${}^nP_n\, ax^{n-n}$, ${}^nP_{n+1}\, ax^{n-(n+1)}$, ----- .
and gives the Christ-Obimba geometric progression series :
$ax^n + {}^nP_1\, ax^{n-1} + {}^nP_2\, ax^{n-2} + {}^nP_3\, ax^{n-3}$ + ------ + ${}^nP_m\, ax^{n-m} + {}^nP_n\, ax^{n-n} + {}^nP_{n+1}\, ax^{n-(n+1)}$ +----- .
whose sum, $S_{m+1} = T_1 \frac{(1-\left[(n-m+1)\frac{1}{x}\right]^{m+1})}{(1-\left[(n-m+1)\frac{1}{x}\right])}$,
is known as the sum of terms of the Christ-Obimba Geometric Progression series in calculus. The Christ-Obimba Geometric Progression series in calculus could be used to obtain all the derivatives, of the parent function, of a finite series, or as many of the derivatives as one would want to obtain, of the parent function, of a finite or infinite series, without calculating individual derivatives, singly.
The formula, $T_i = \frac{T_d}{(n-m+1)\times\frac{1}{x}} + C,$
could be used in calculus of integration, for determining integrals of functions and is known a Christ-Obimba, Calculus Rule 3.

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